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# DEFORMATION MODEL FOR EUROPE: APPLICATION OF THE LEAST-SQUARE COLLOCATION

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AMBRUS KENYERES<sup>3</sup>, ELMAR BROCKMANN<sup>4</sup>, SIMON LUTZ<sup>4</sup>

1 – LANTMÄTERIET

2 – ROYAL OBSERVATORY OF BELGIUM

3 – SATELLITE GEODETIC OBSERVATORY HUNGARY

4 – FEDERAL OFFICE OF TOPOGRAPHY SWISSTOPO

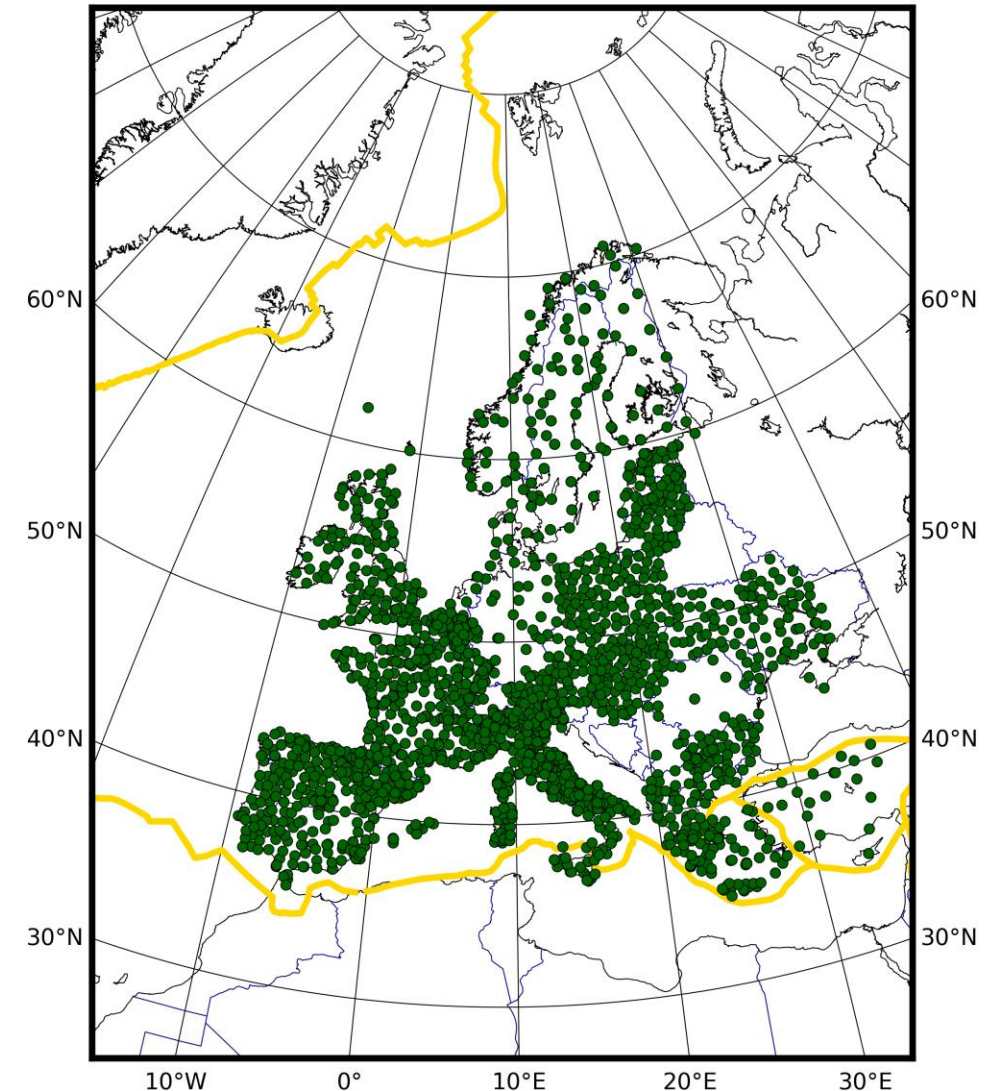
LANTMÄTERIET

# INTRODUCTION

- One of the goals of EUREF (Regional Reference Frame Sub-Commission for Europe) is the development of a deformation model for Europe
  - Estimation of a dense velocity grid
  - Using GNSS-based station velocity solutions

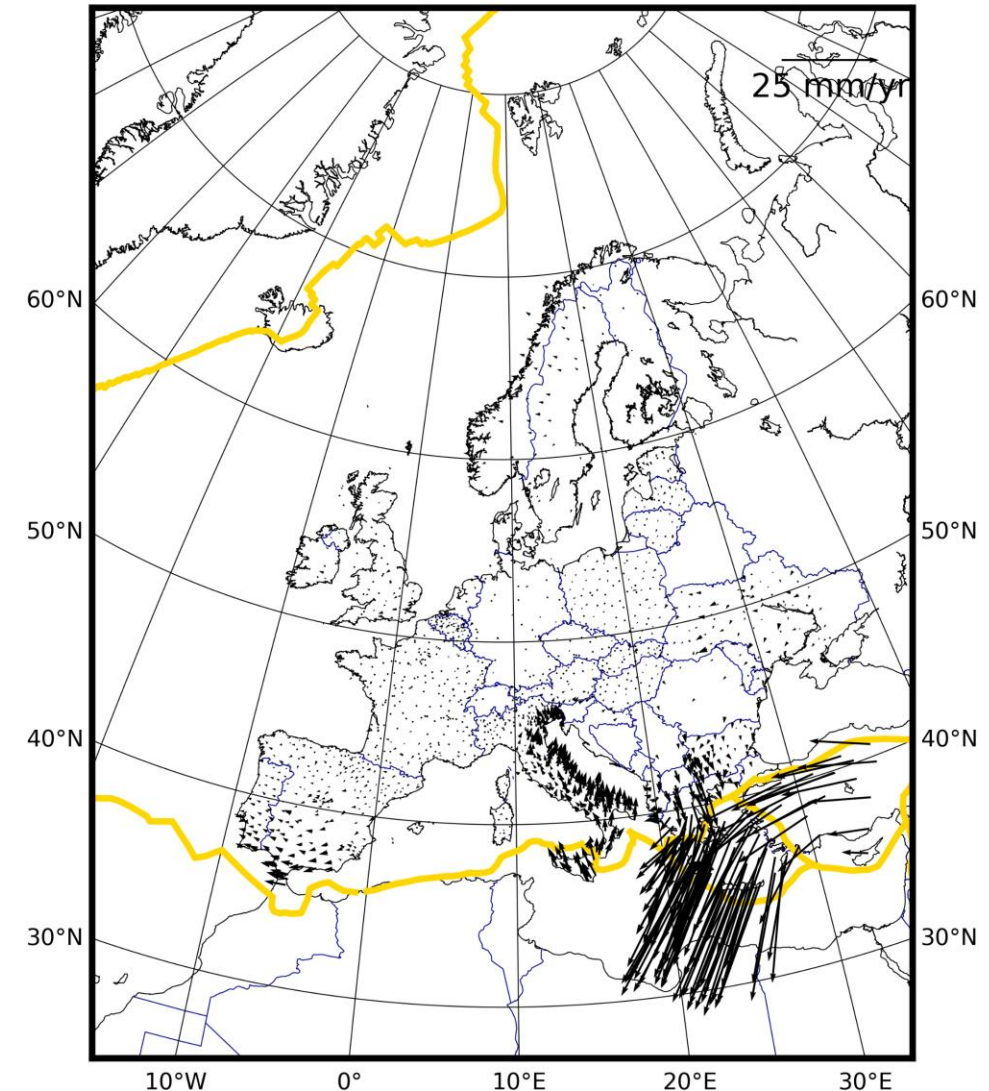
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  - For example: “EPN densification” by Kenyeres
    - Regional weekly GNSS solutions (SINEX format) combined to weekly solutions, and station velocities estimated by rigorous stacking of the combined weekly solutions in the CATREF software
    - Data cleaning is an important part of the process and stations with unrealistic velocities (mostly due to short time series in the 2 – 3 years domain) are removed
    - More information: [http://epncb.oma.be/\\_densification/](http://epncb.oma.be/_densification/)
    - Dataset “EDVI4\_ENEU\_v3.filt” from August 8<sup>th</sup>, 2018, is used in the following
    - Dataset is in ETRF2000

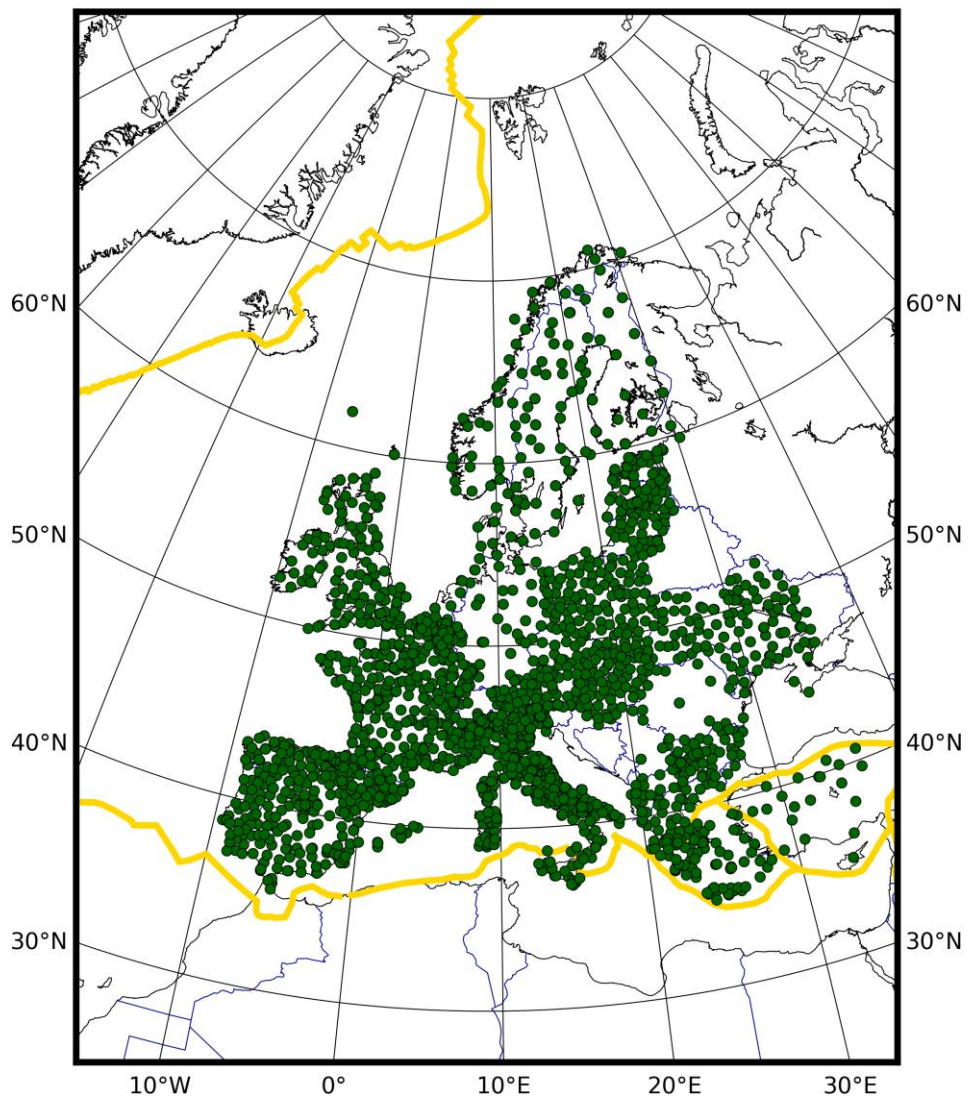


# INTRODUCTION

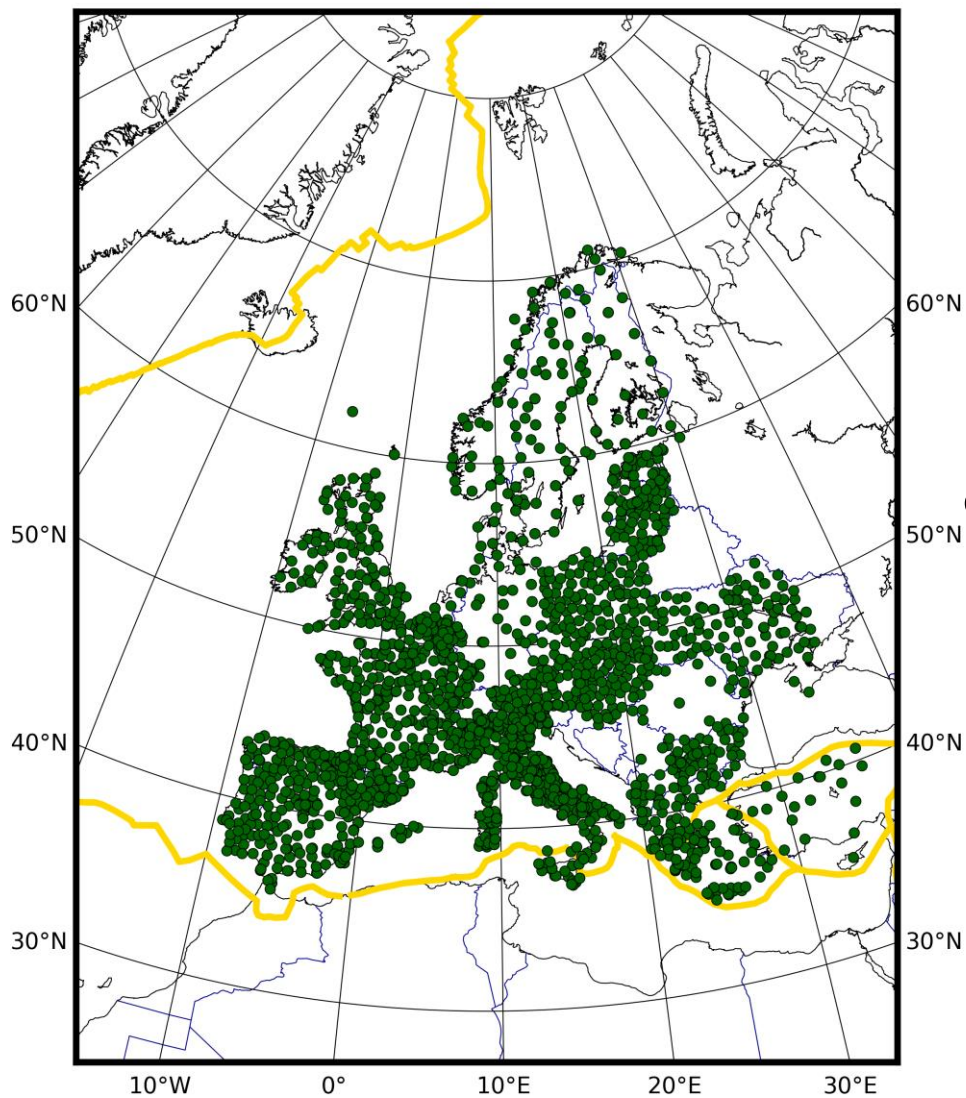
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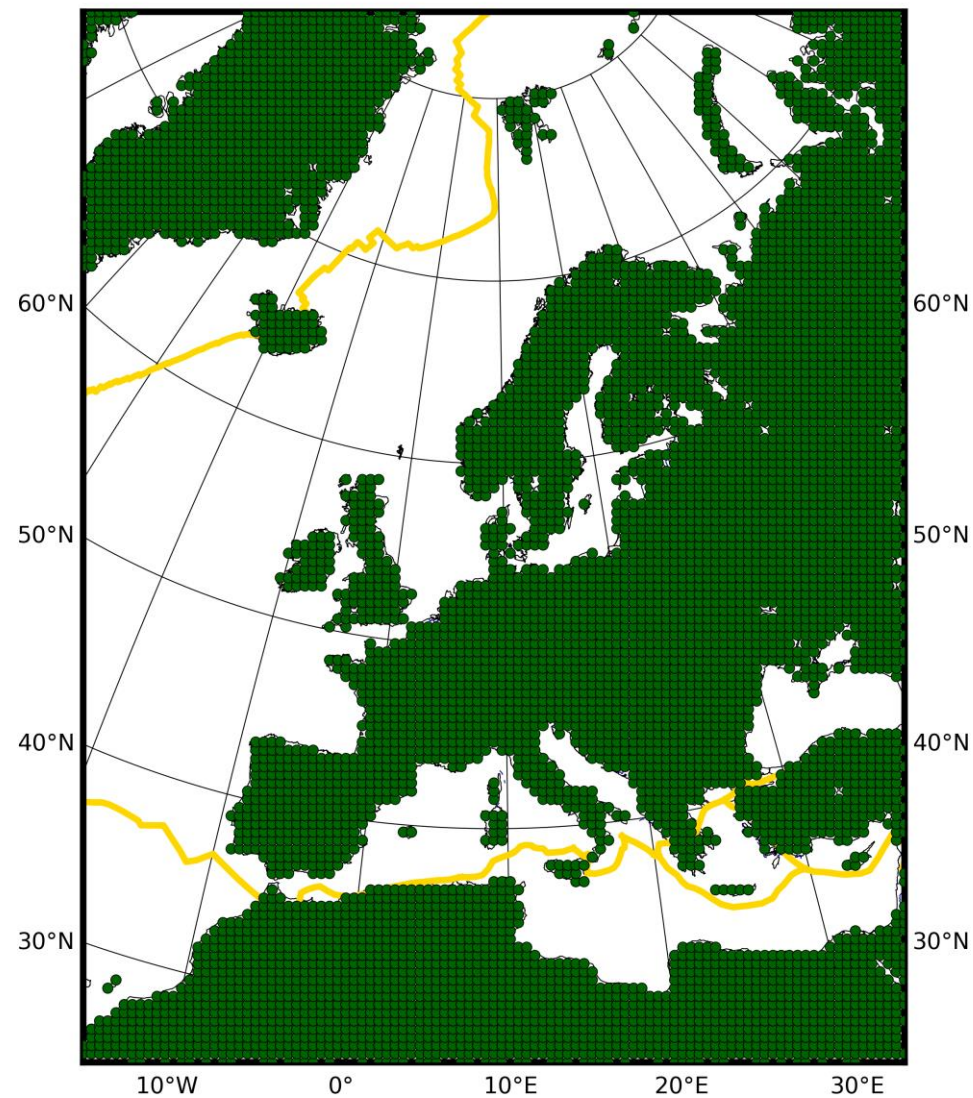
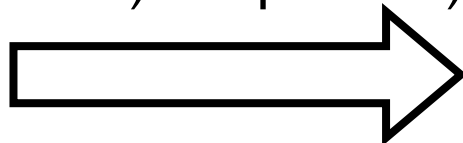
# INTRODUCTION



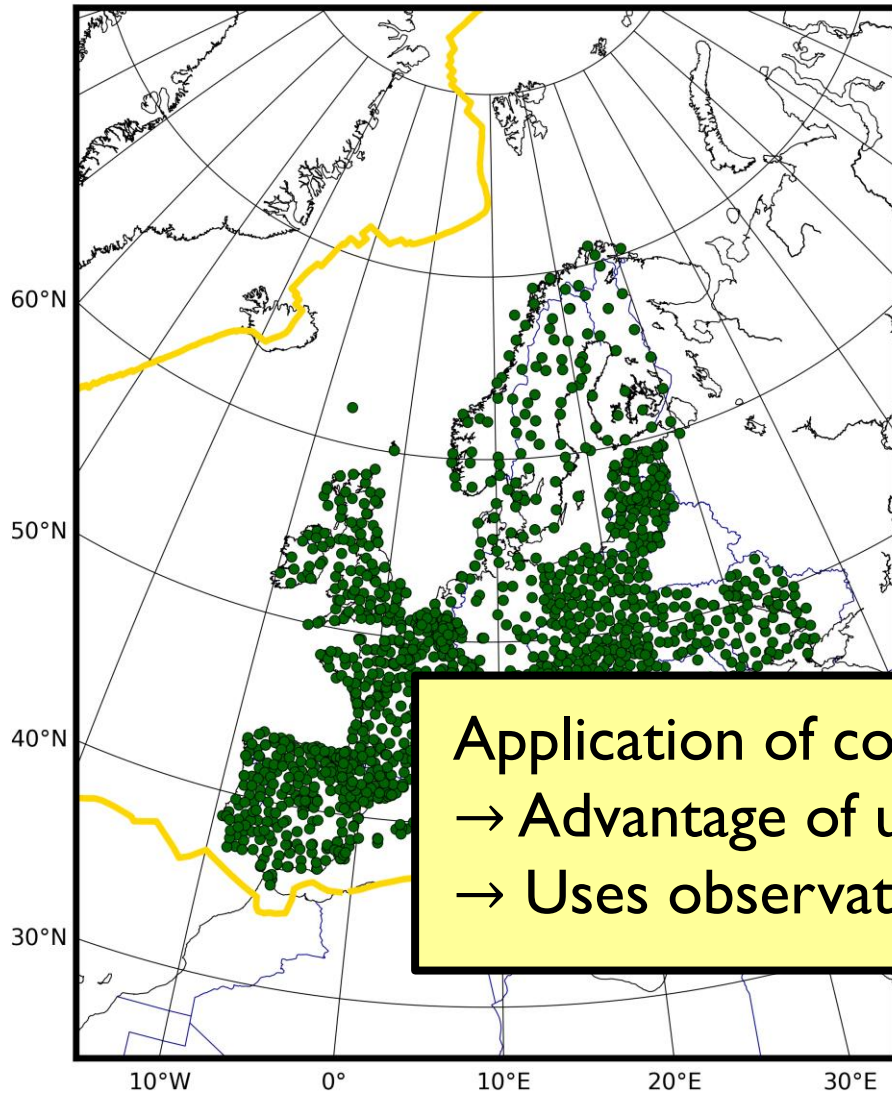
# INTRODUCTION



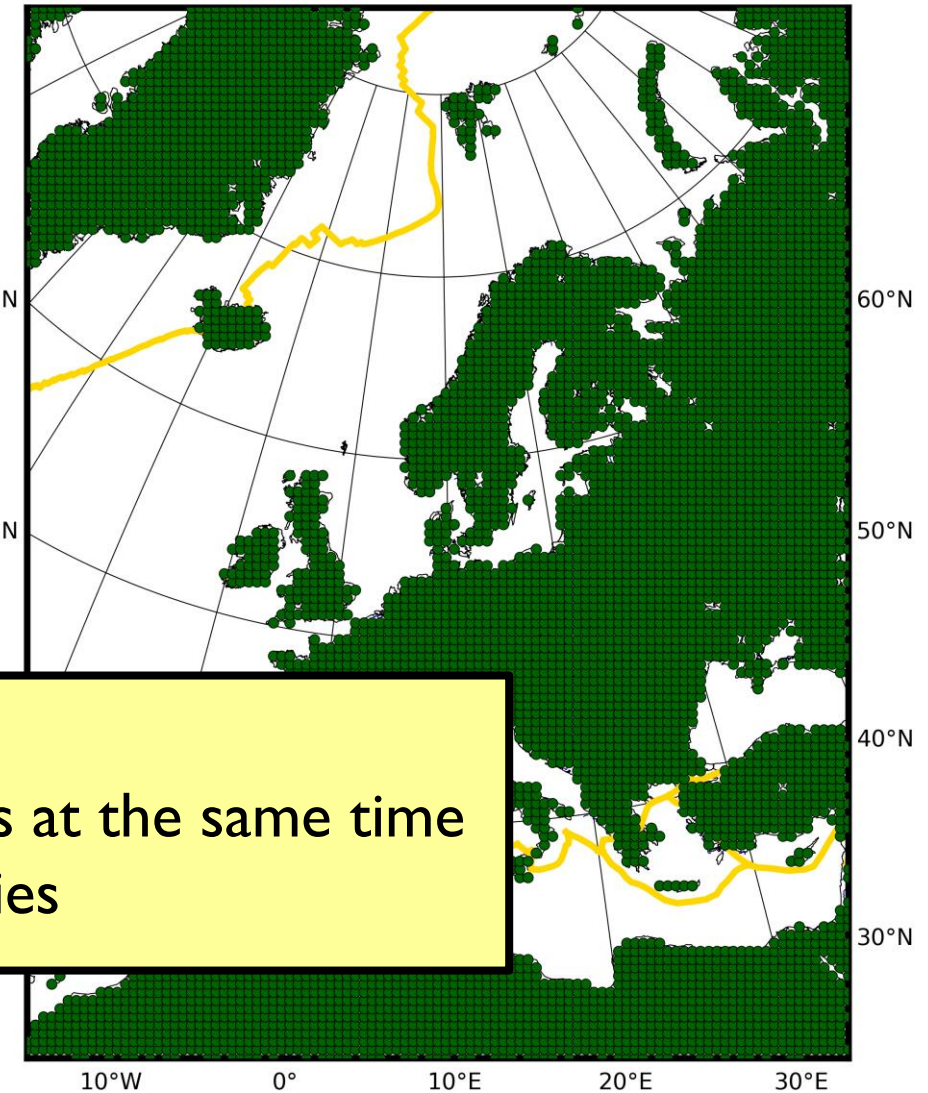
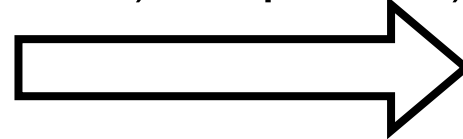
Interpolation to  
50 km x 50 km grid  
(other grids (e.g.,  
denser) are possible)



# INTRODUCTION



Interpolation to  
50 km x 50 km grid  
(other grids (e.g.,  
denser) are possible)



Application of collocation methodology  
→ Advantage of using several observations at the same time  
→ Uses observations including uncertainties

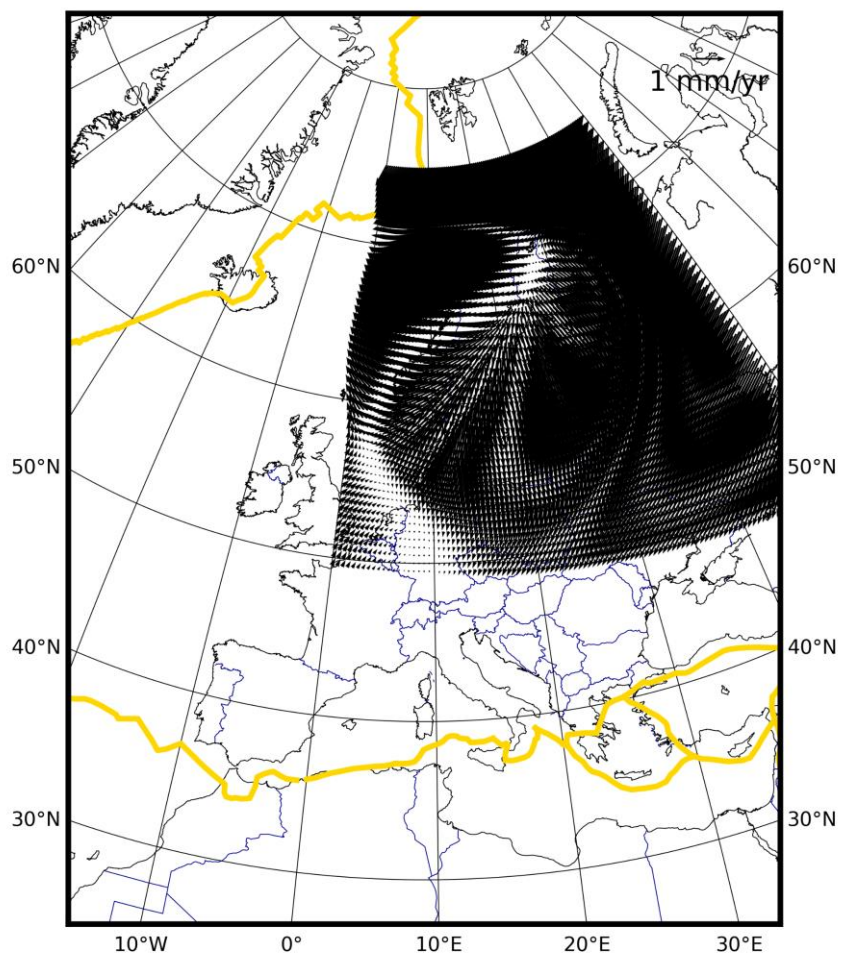
# COLLOCATION (SHORT SUMMARY)

- Velocity data are filtered and interpolated (prediction) using least-square collocation (LSC, based on Moritz, 1980)
  - $l = s + n$
  - $l$  – observations
  - $s$  – signals
  - $n$  – noise
- Signal and noise can be separated and the signal and the corresponding uncertainty can be obtained at observation points or new points
- Calculation involves the estimation of covariance matrices → depends only on the distance between the points and the choice of the covariance function  
→  $C_0$  (signal covariance) and  $d_0$  (correlation length) have to be determined
- All known information should be reduced from the observations before covariance calculation and collocation are applied (e.g., background model, mean value)  
→ added afterwards again

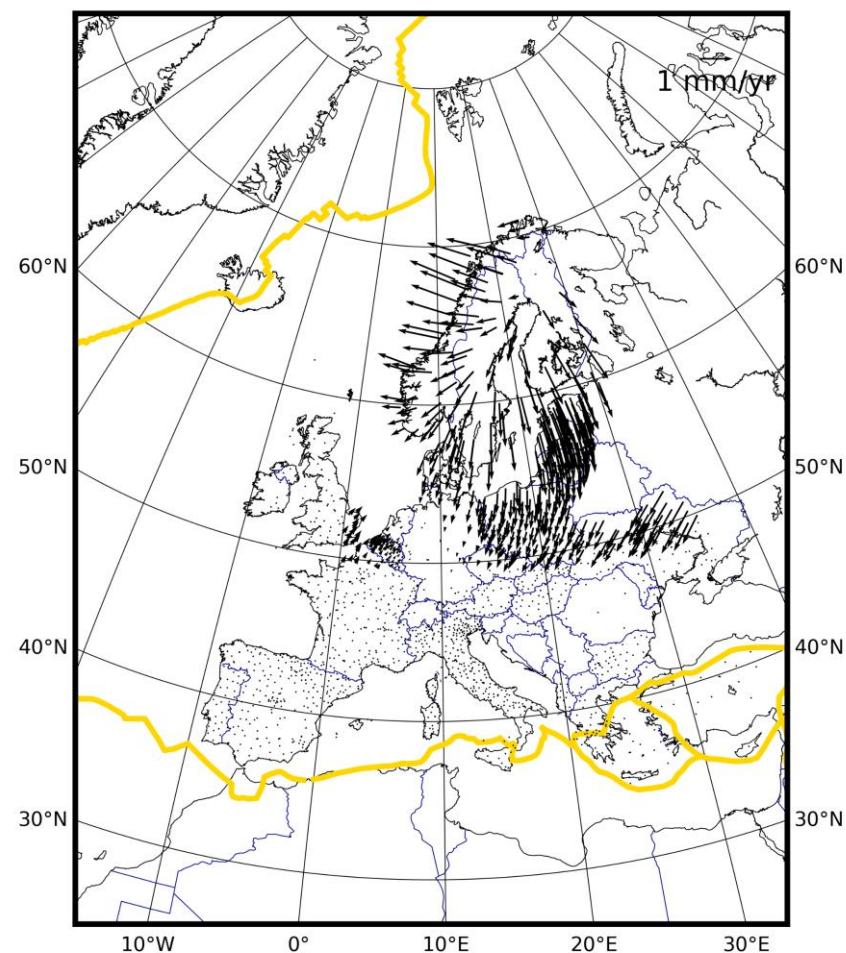
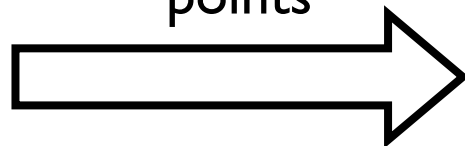


# BACKGROUND MODEL

Background model is reduced from observational data → theoretical GIA (Glacial Isostatic Adjustment) model rotated into ETRF2000 (same reference frame as the GNSS data)

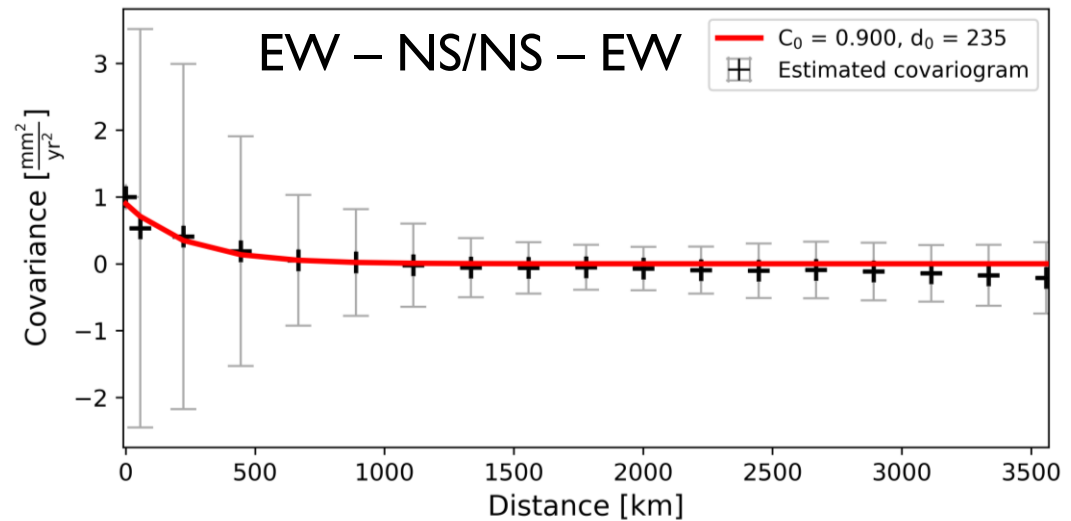
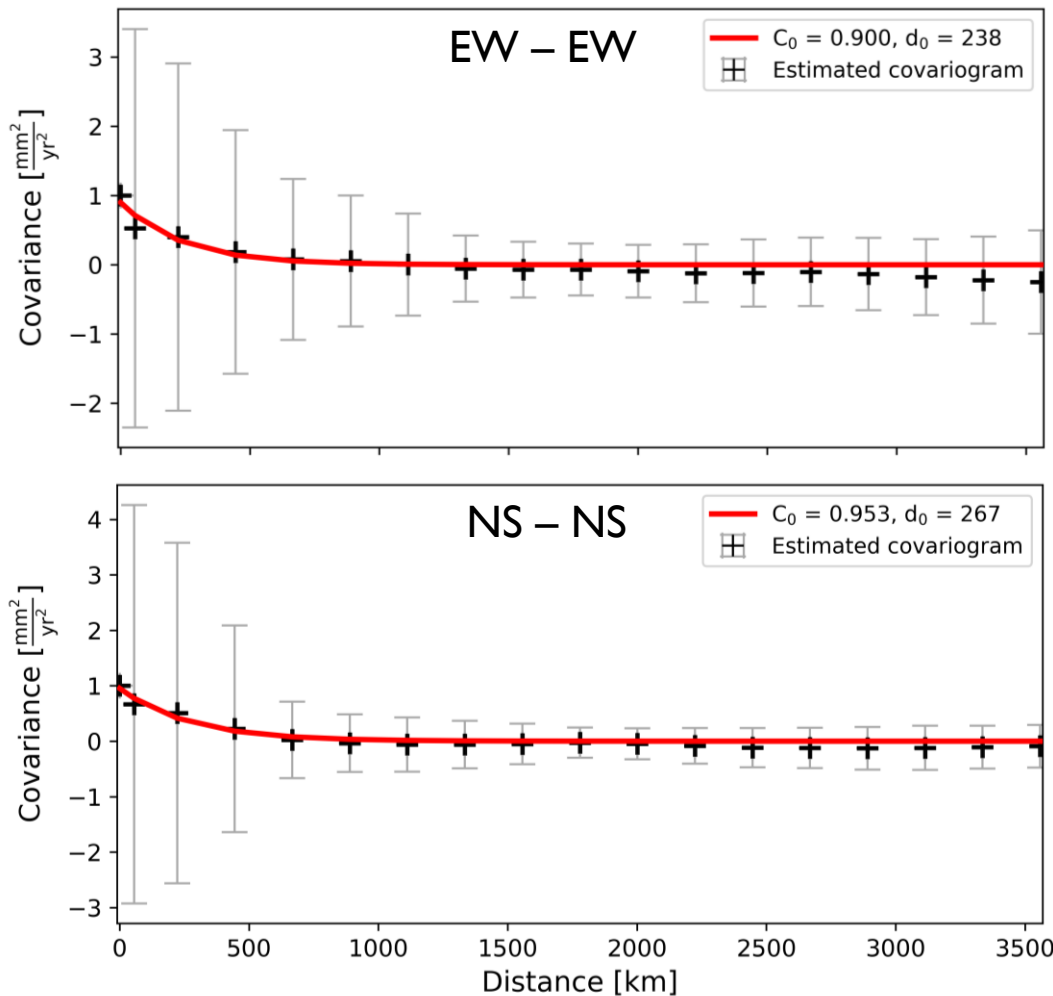


interpolated to  
observational  
points



# COVARIANCE FUNCTION

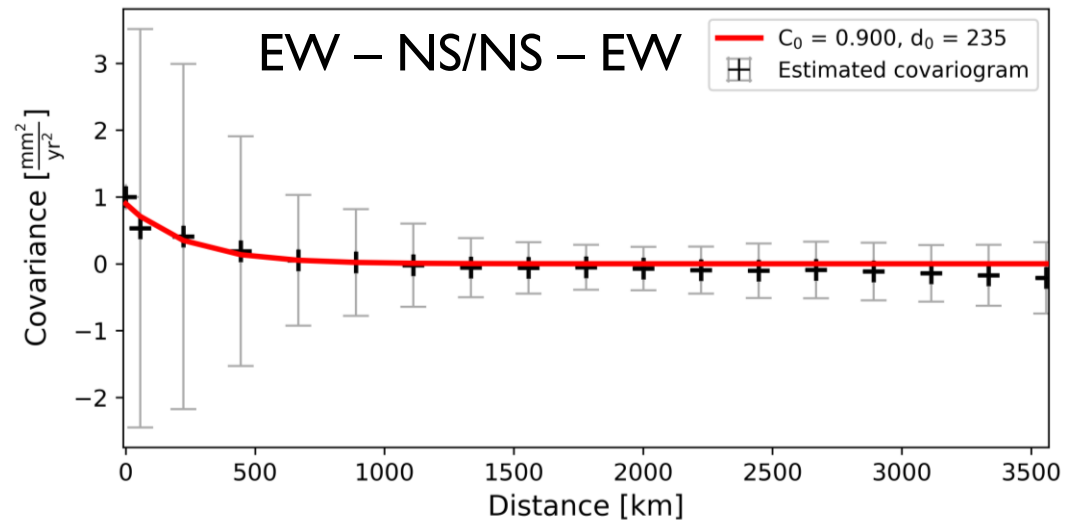
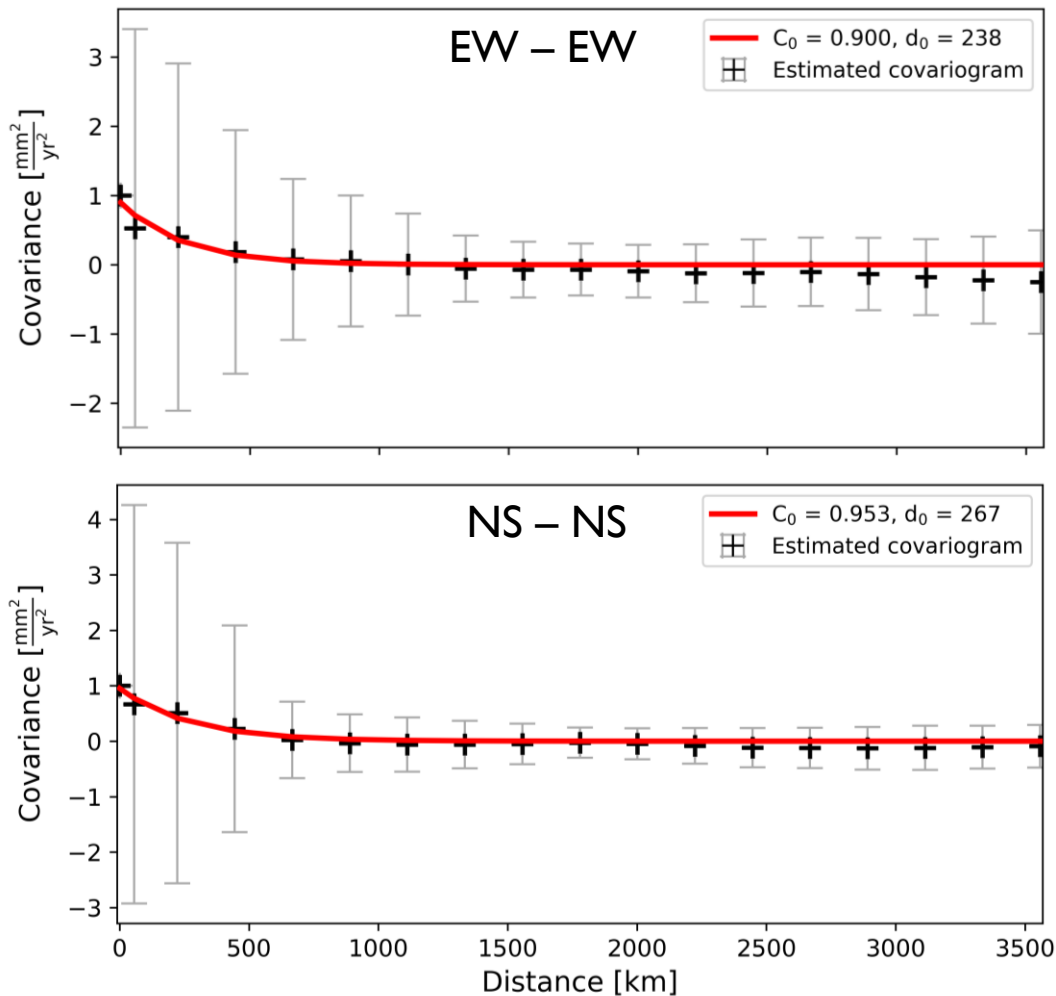
Gauss-Markov 1st order used:  $K(d) = C_0 \cdot e^{-d/d_0}$  (covariances are normed)



	$C_0 [mm^2/yr^2]$	$d_0 [km]$
EW – EW	$0.900 \pm 0.111$	$238 \pm 73$
NS – NS	$0.953 \pm 0.076$	$267 \pm 51$
EW – NS NS – EW	$0.900 \pm 0.095$	$235 \pm 61$

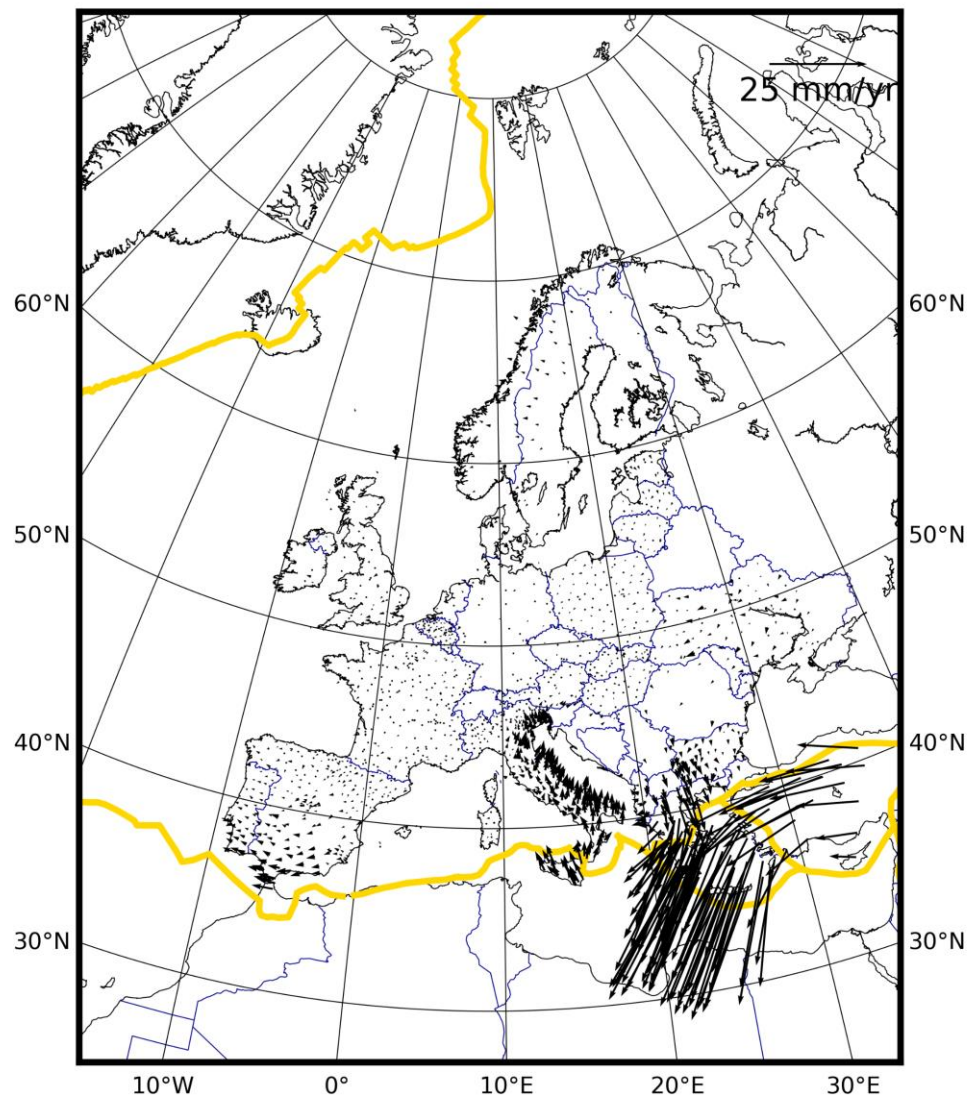
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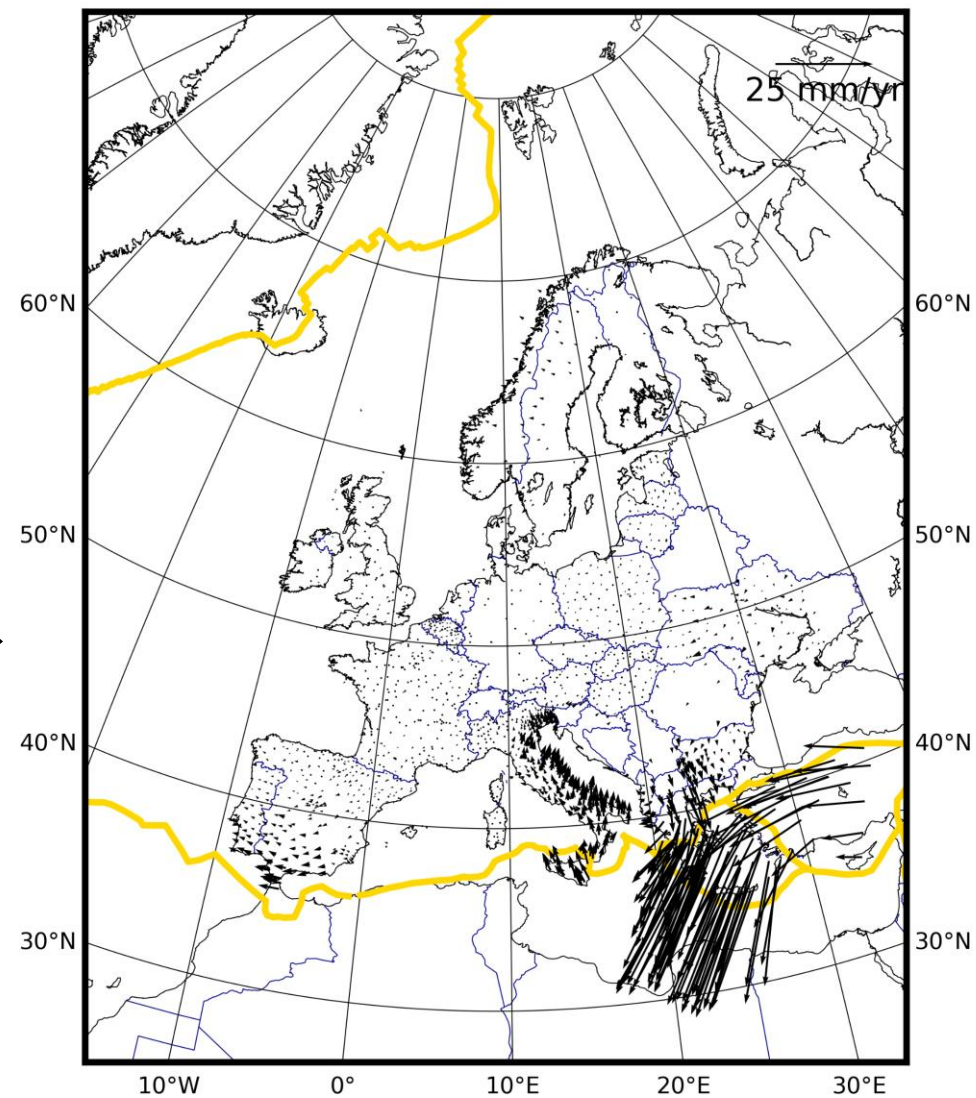
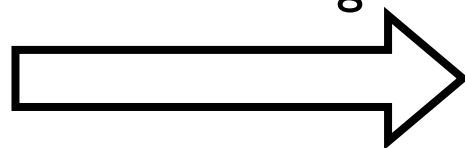


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<b>Final</b>	<b>1.0</b>	<b>250</b>

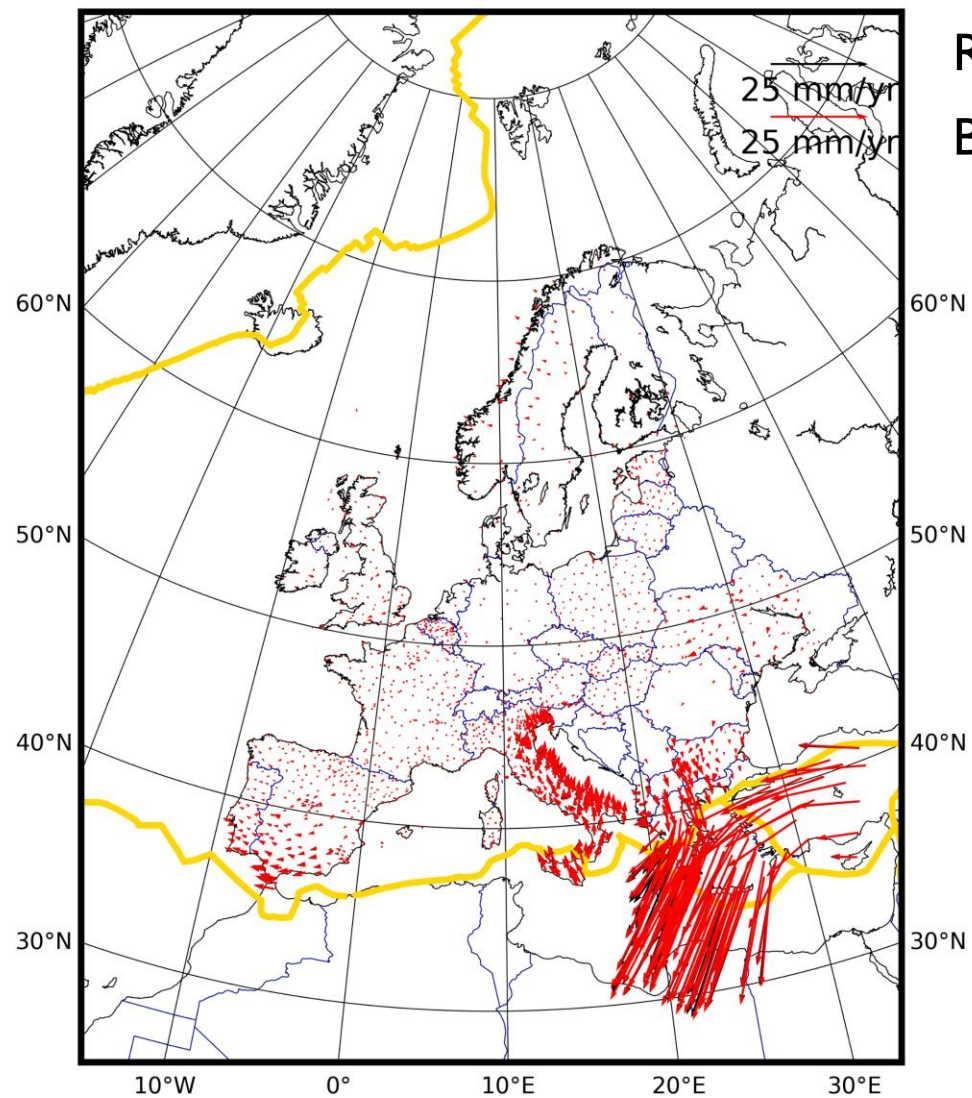
# COLLOCATION – FILTERING



Separation of the  
observation into a  
signal and noise  
component  
→ Filtering



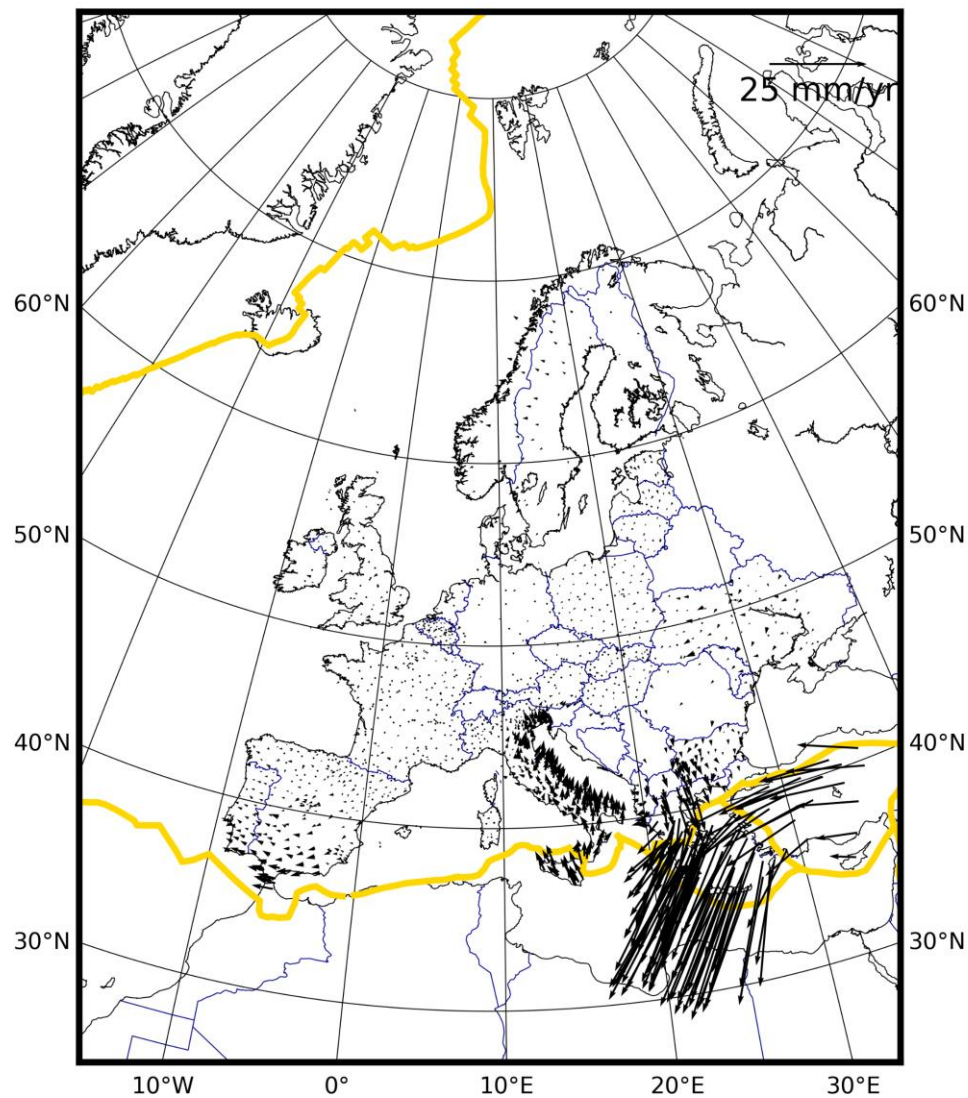
# COLLOCATION – FILTERING



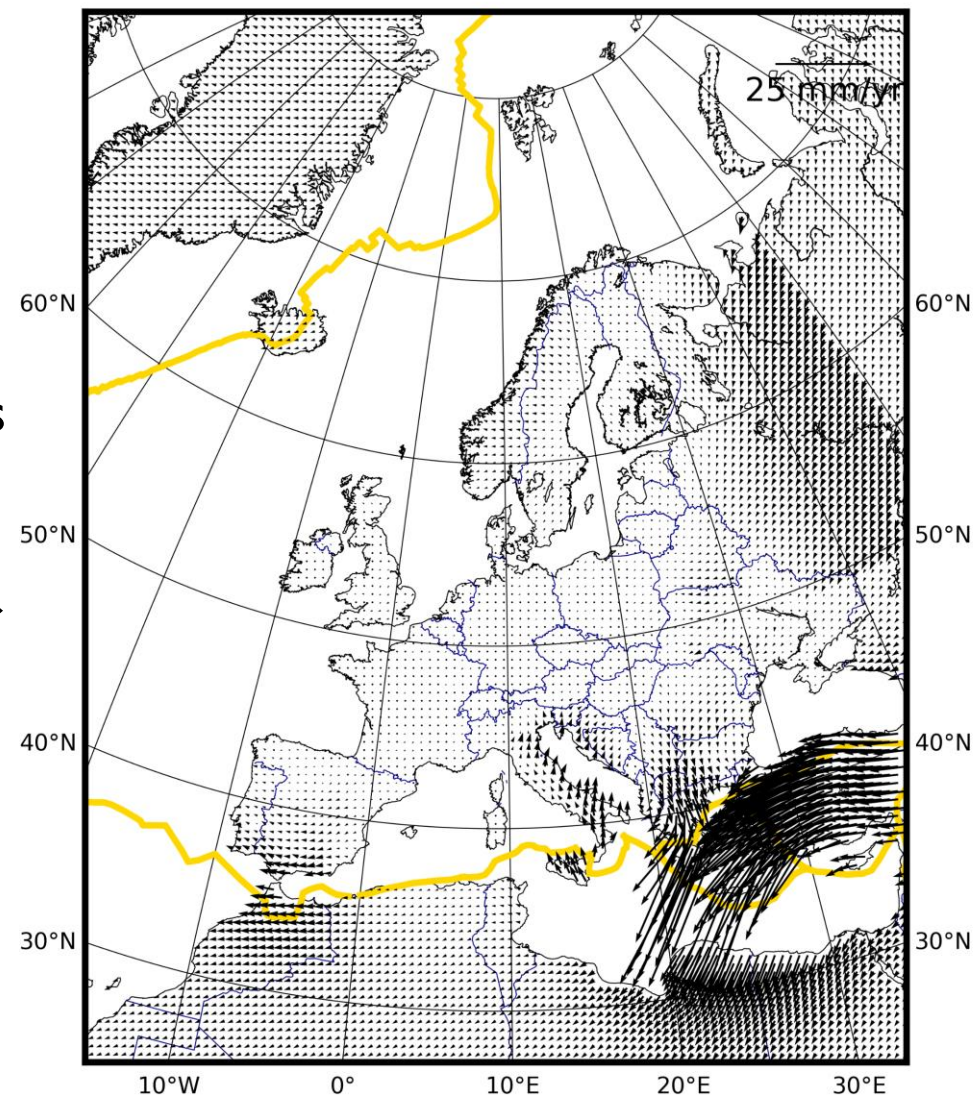
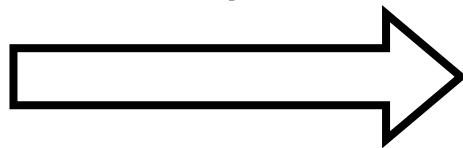
Red – original data

Black – collocated (filtered) data

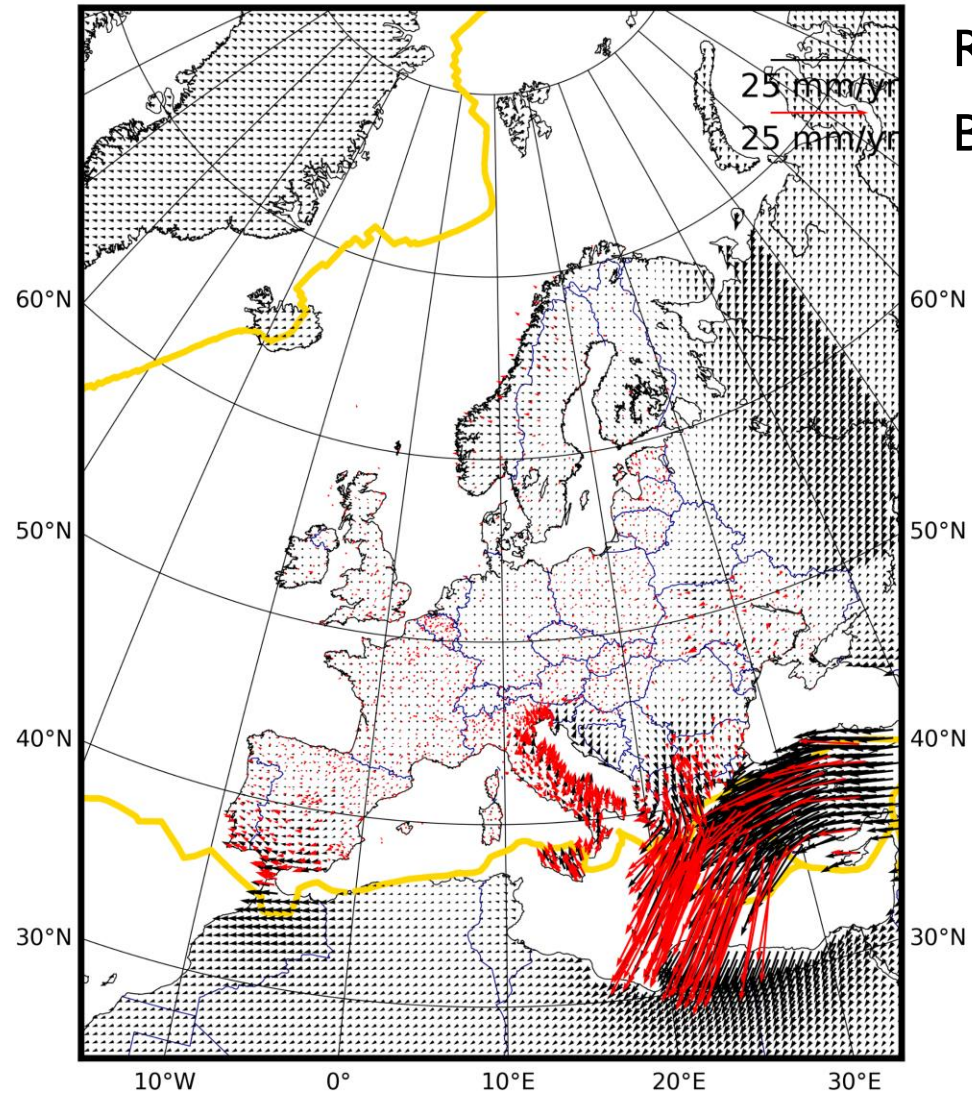
# COLLOCATION – INTERPOLATION



Prediction of the  
signal at new points  
→ Interpolation



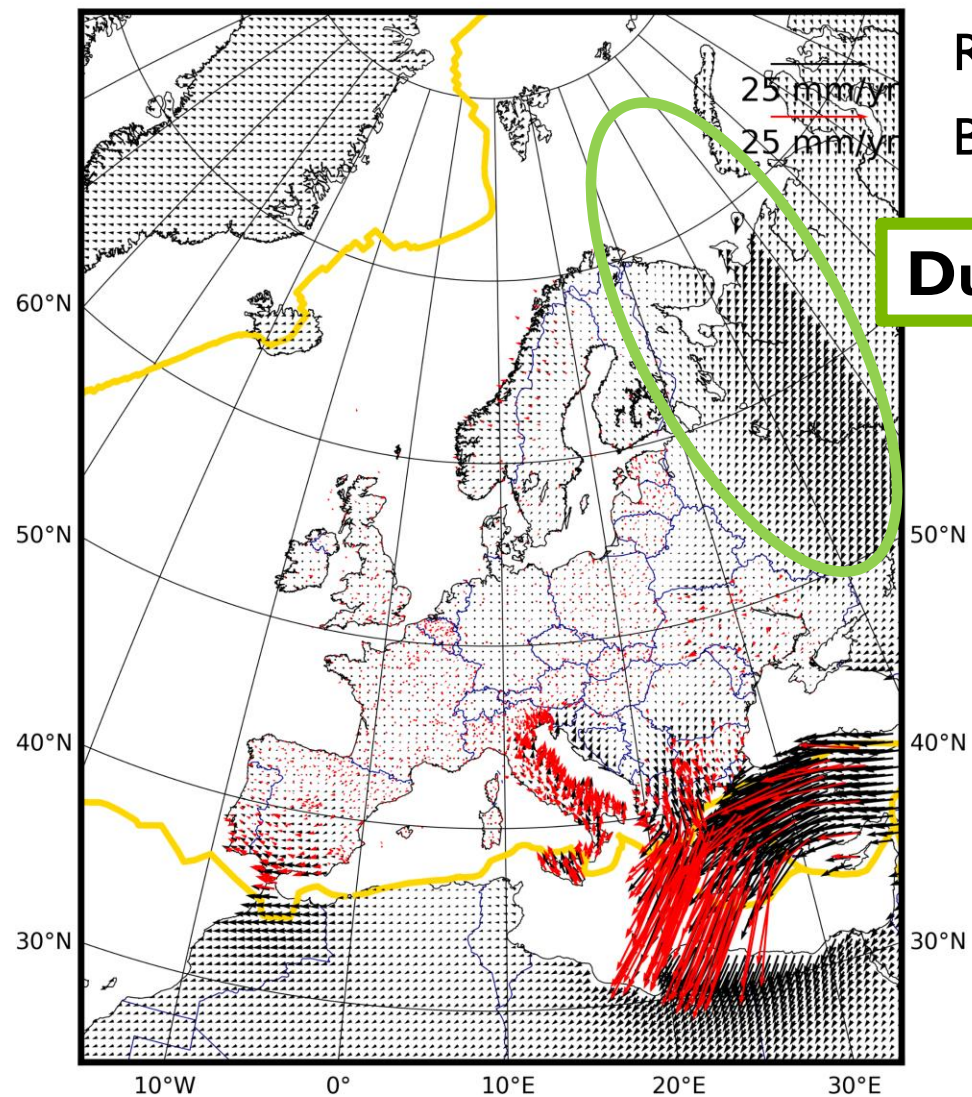
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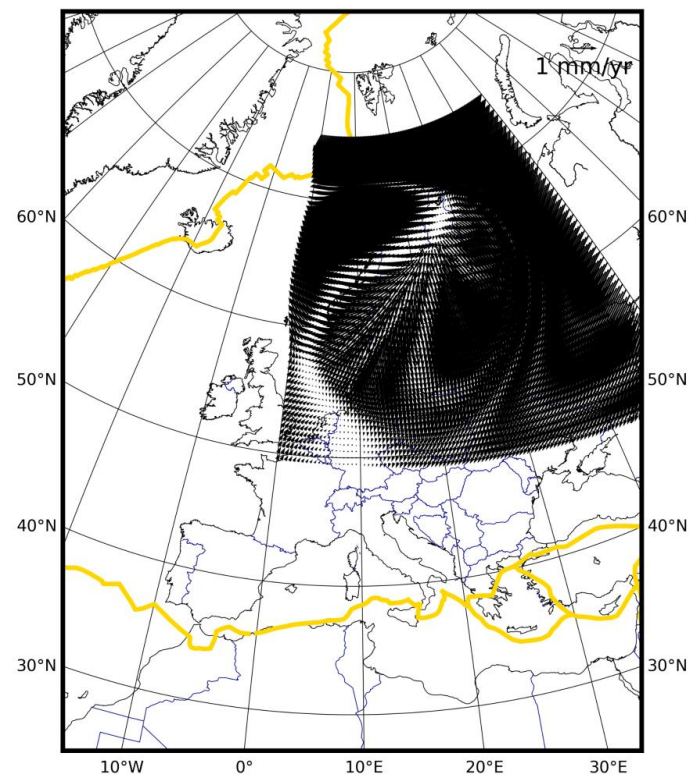
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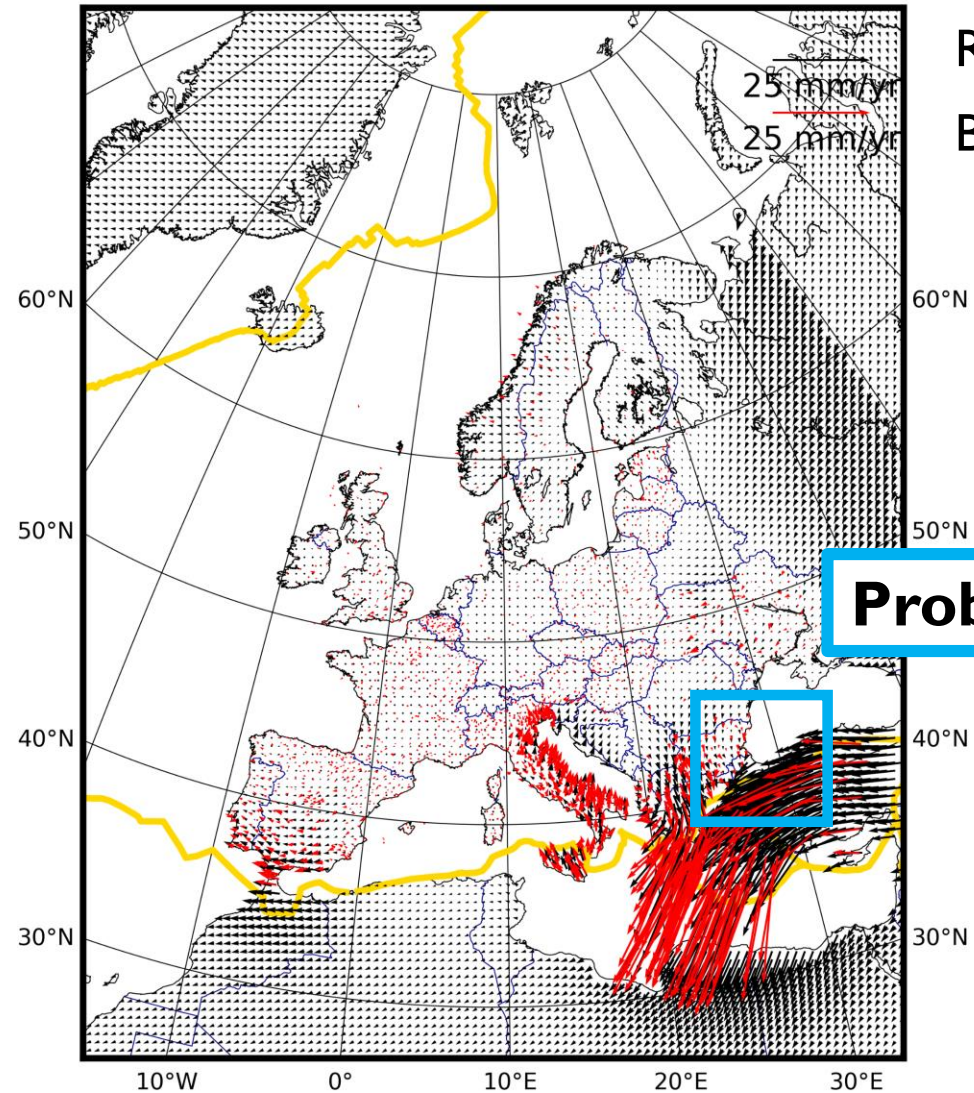
Black – collocated (filtered) data

**Due to background model**





# COLLOCATION – INTERPOLATION



Red – original data

Black – collocated (filtered) data

**Problem along plate boundaries**

# COLLOCATION WITH PLATE BOUNDARIES

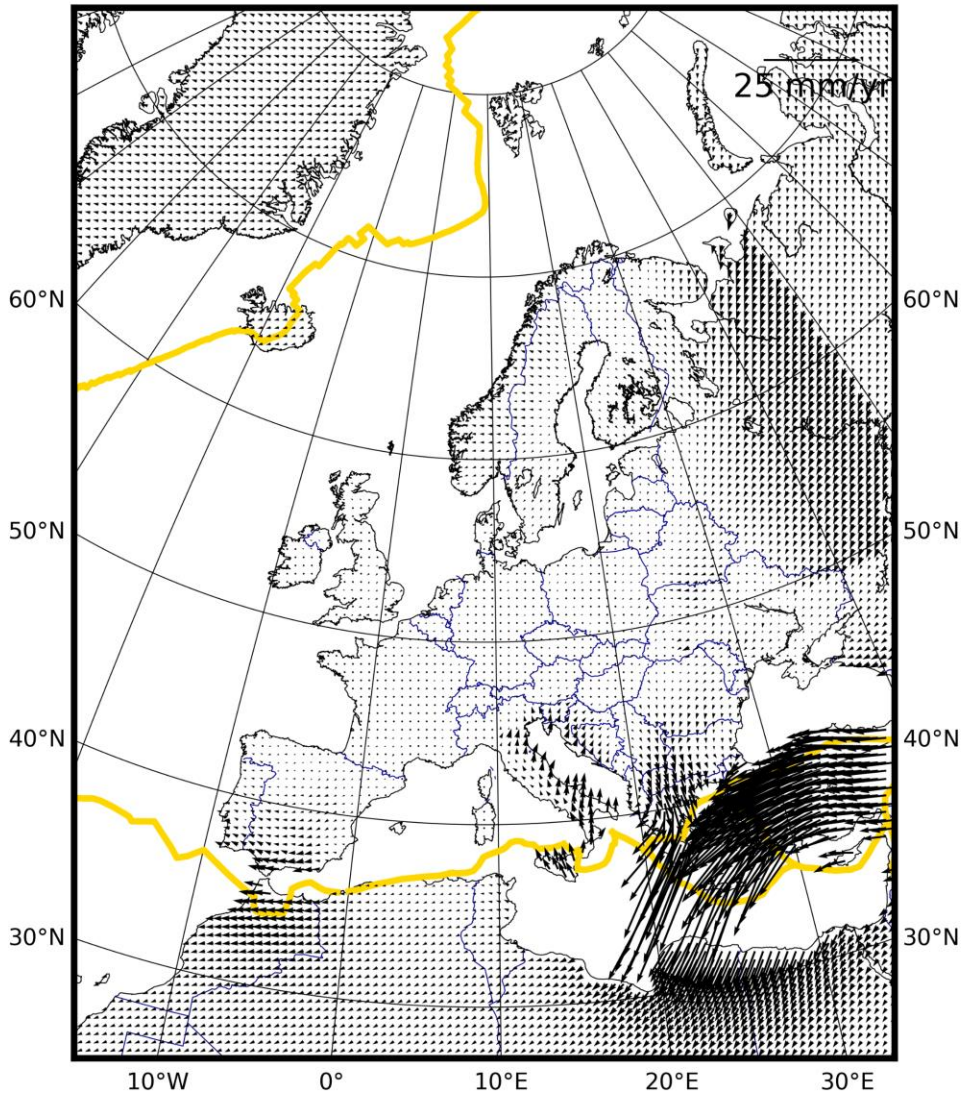
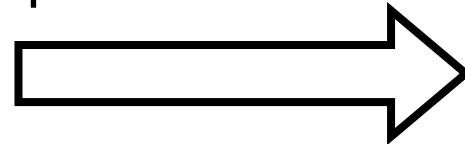
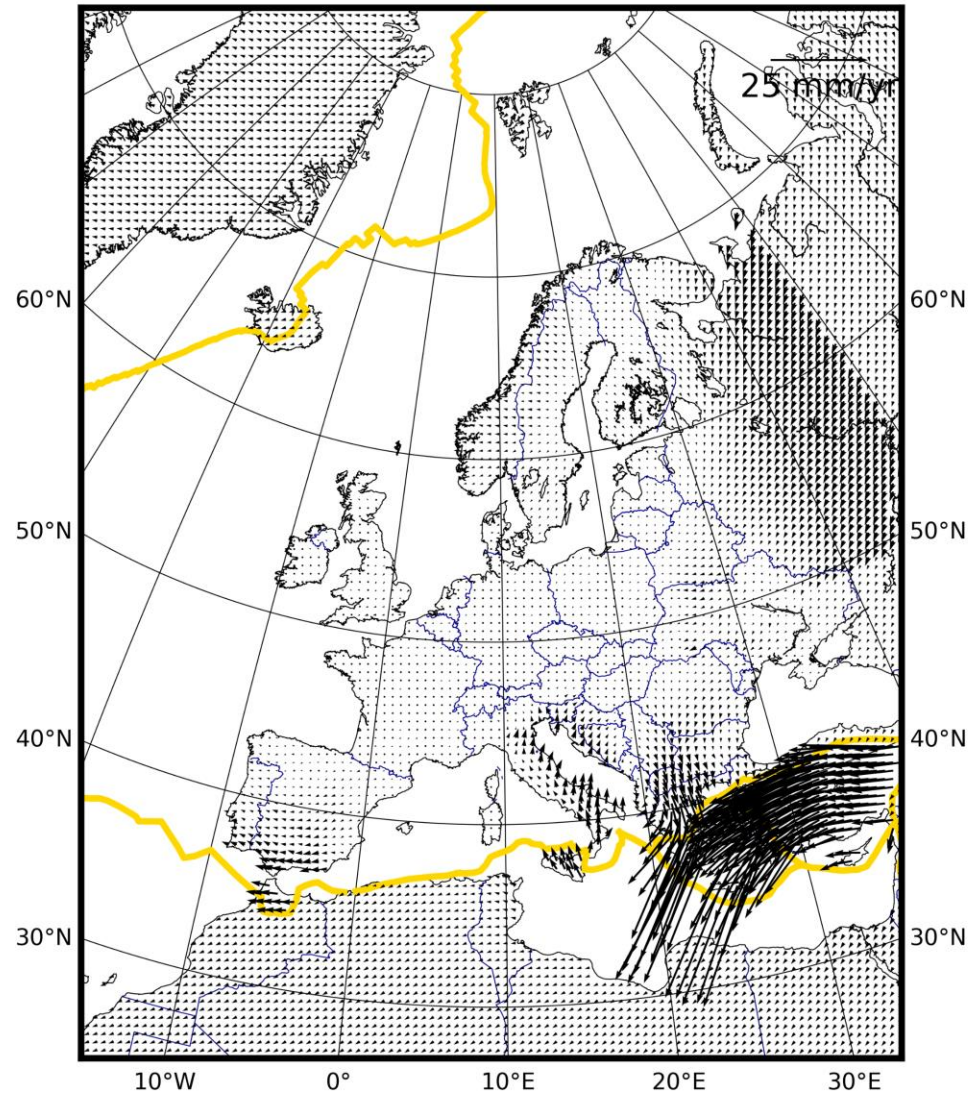


Plate boundaries included  
 → distance between stations on different plates is increased



→ but distance for stations on the same plate is kept the same



# COLLOCATION WITH PLATE BOUNDARIES

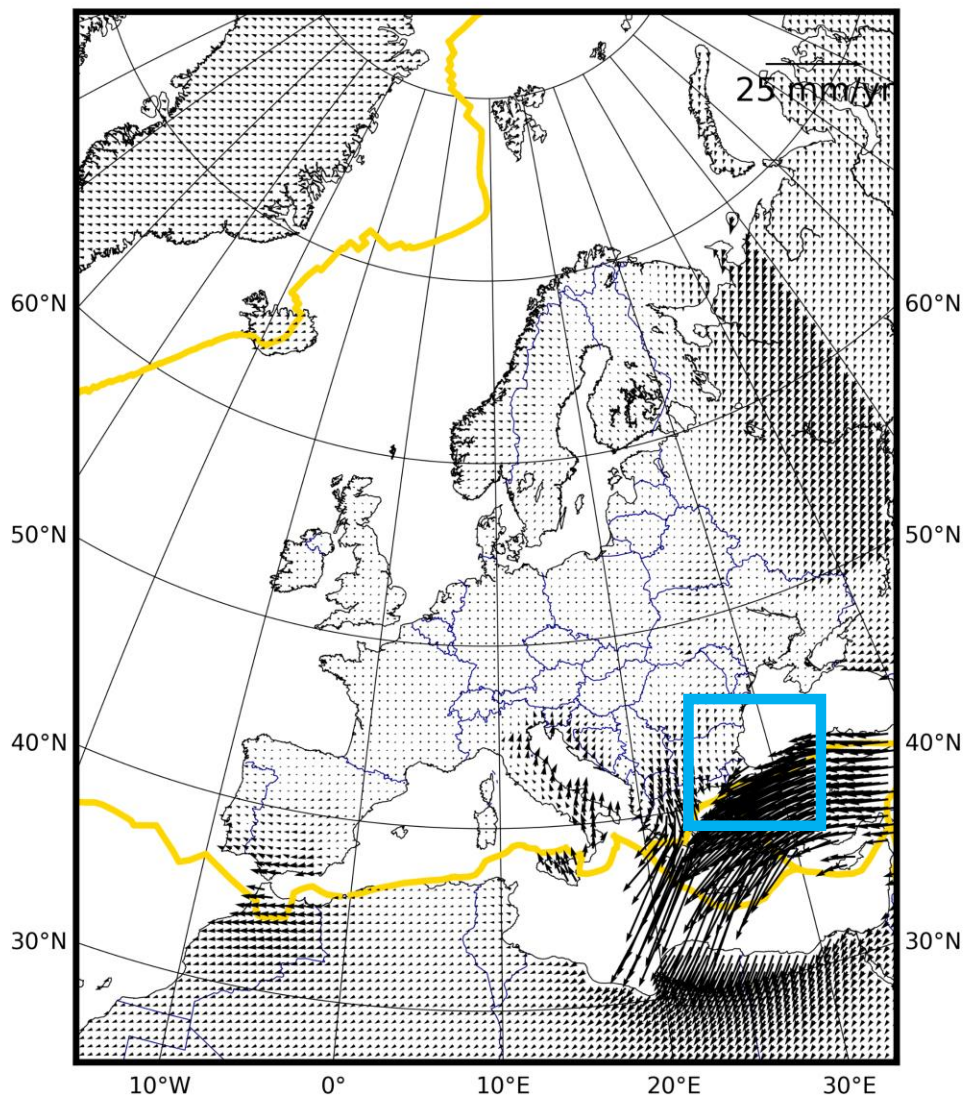
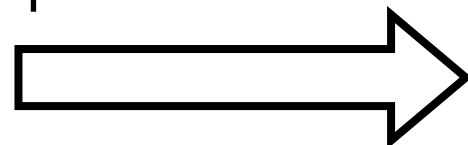
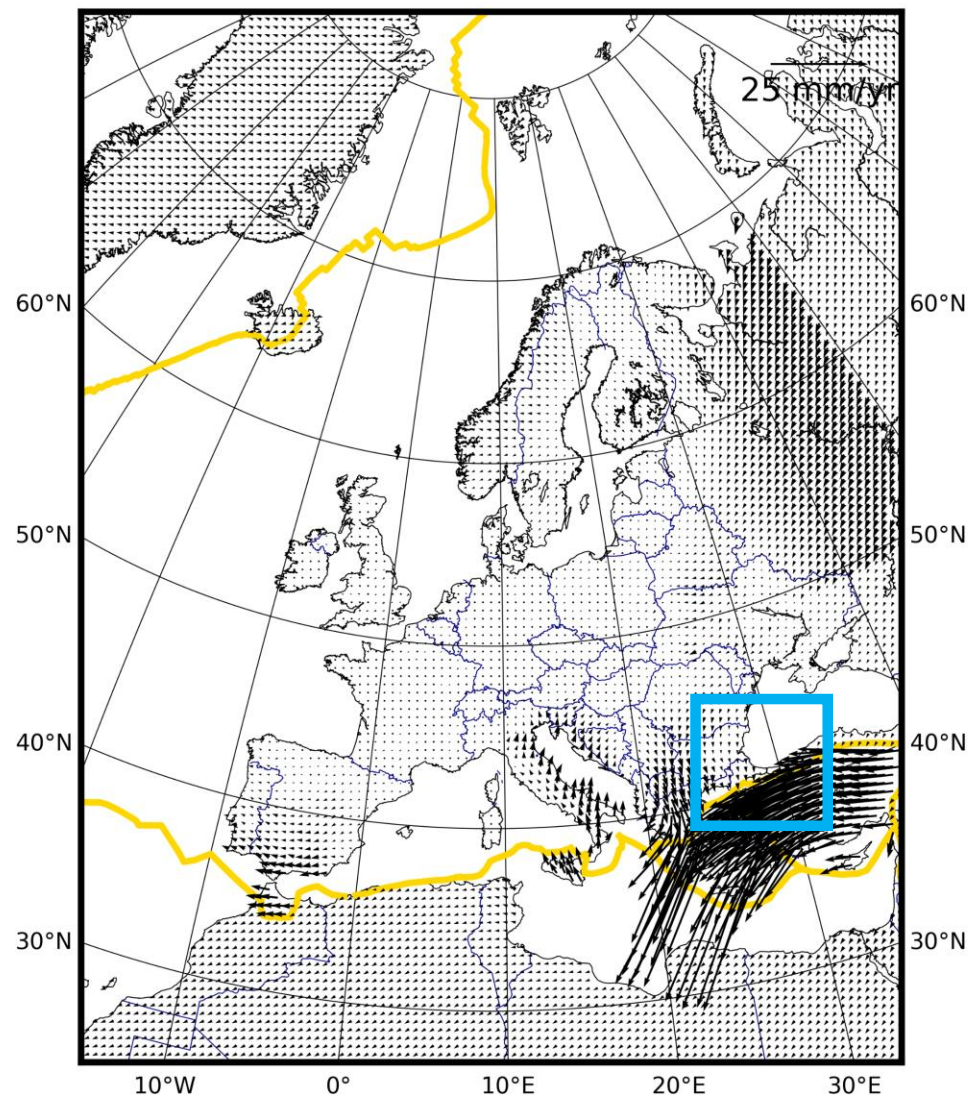


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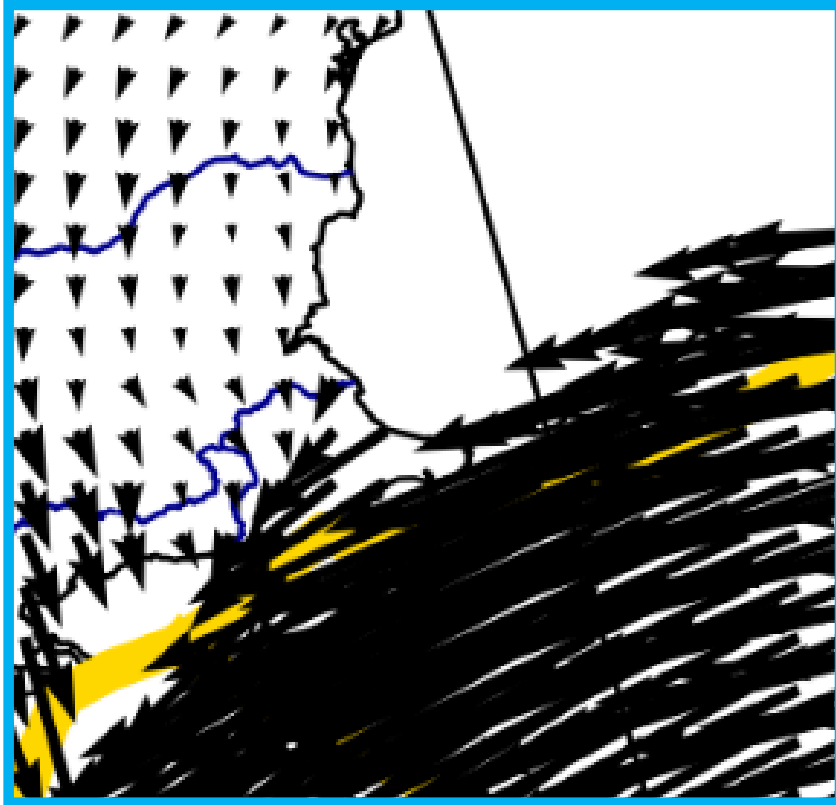


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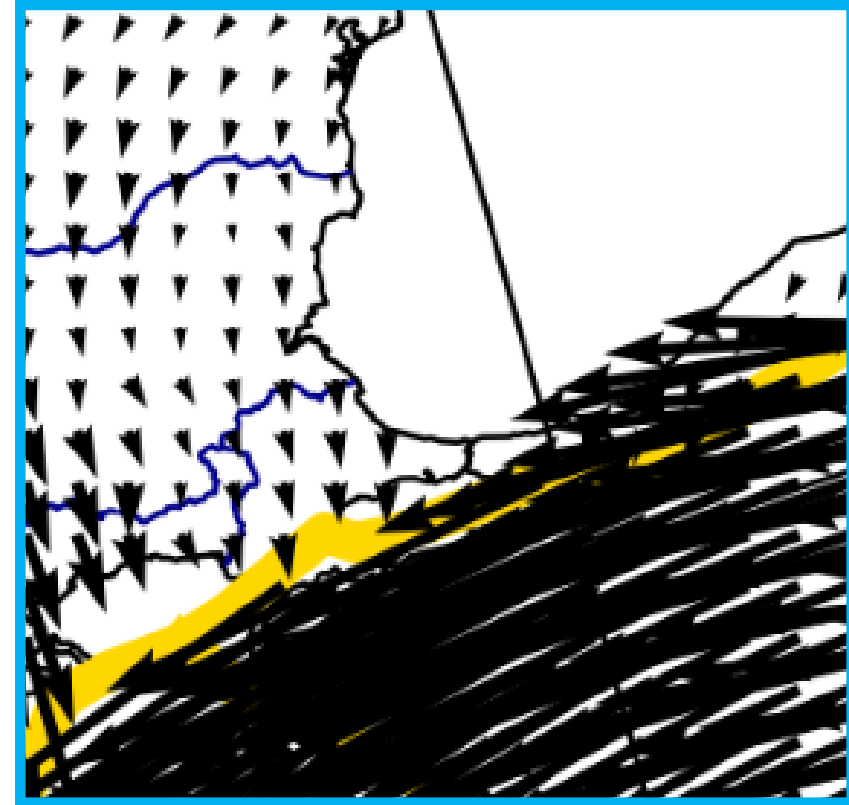


# COLLOCATION WITH PLATE BOUNDARIES

Collocation without plate boundaries

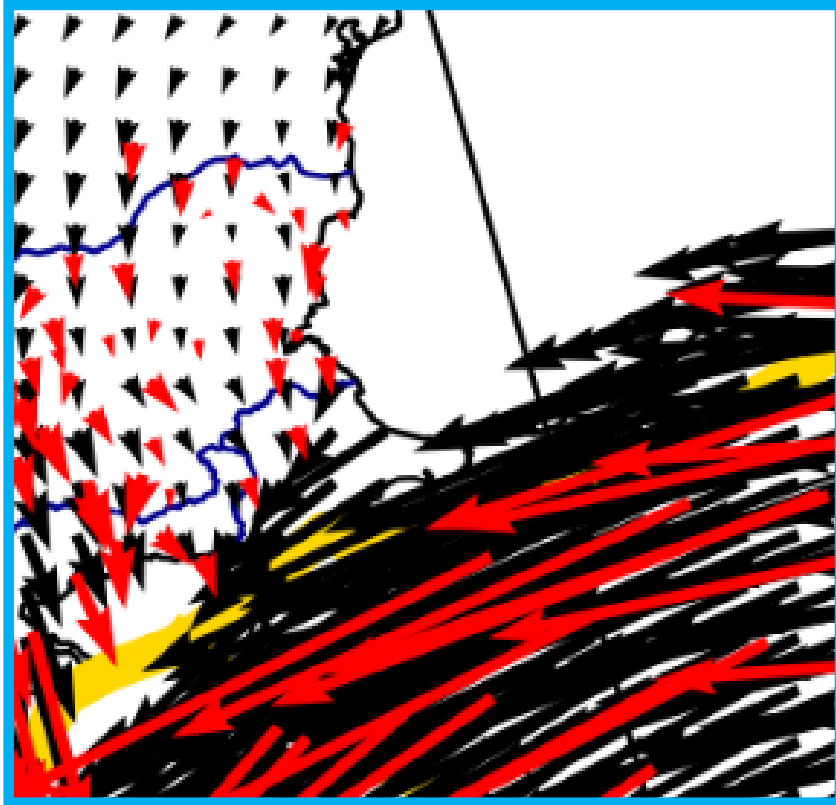


Collocation with plate boundaries

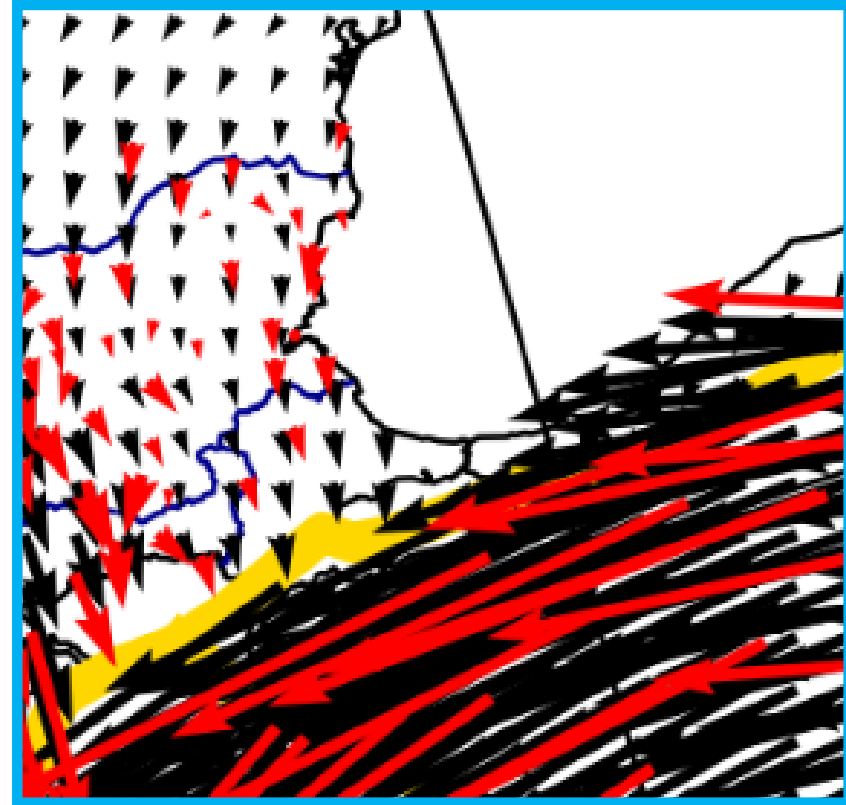


# COLLOCATION WITH PLATE BOUNDARIES

Collocation without plate boundaries



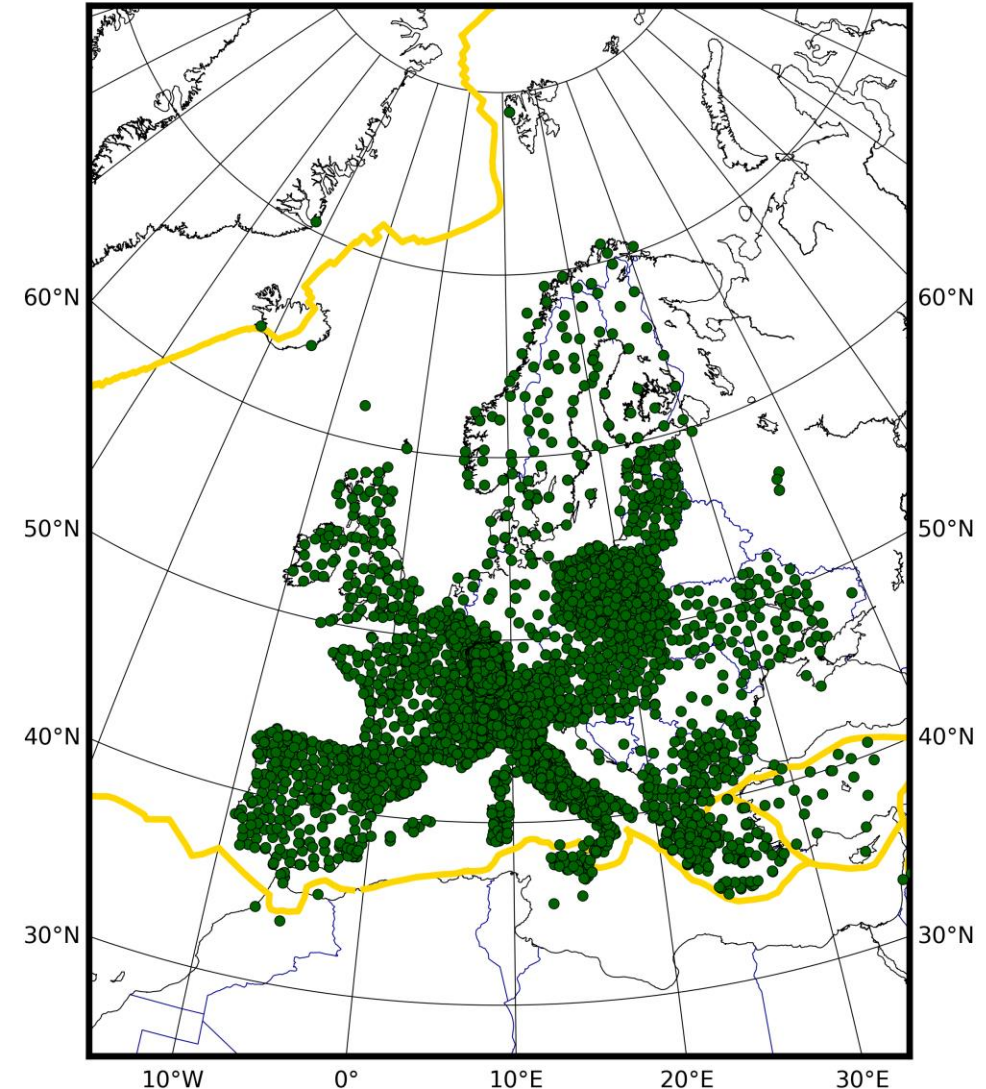
Collocation with plate boundaries



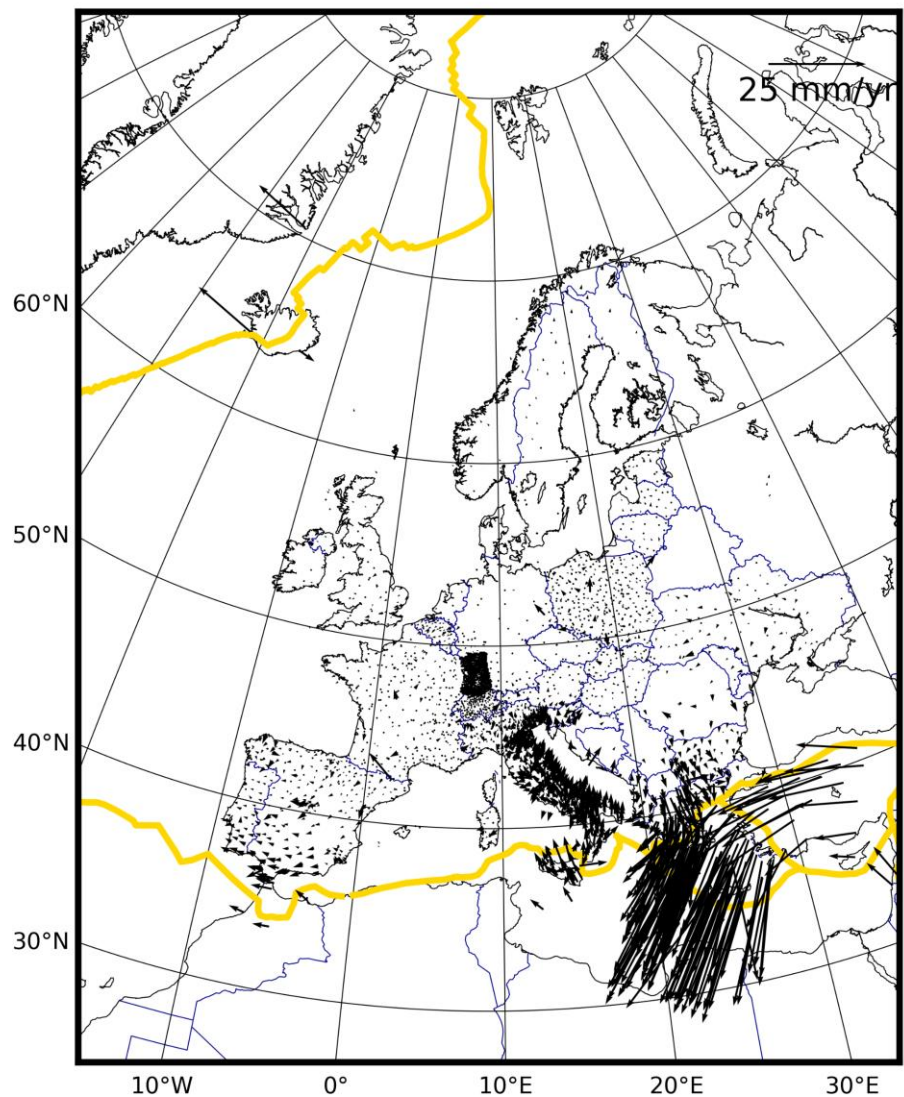
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# COLLOCATION – EU DENSE VELOCITIES

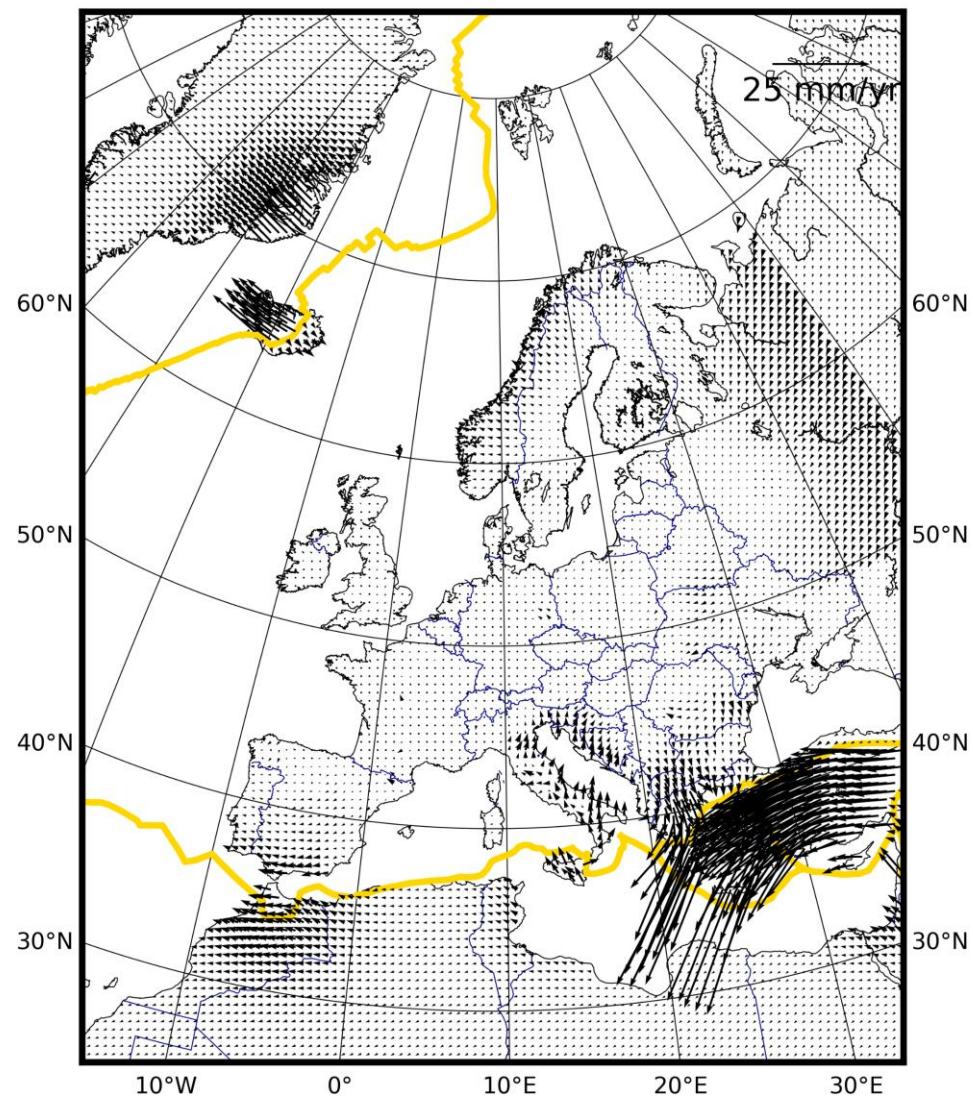
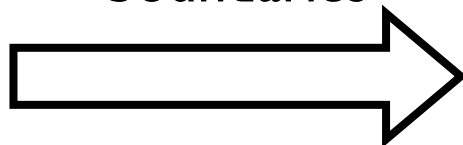
- A different dataset can be used as well:
  - “EU Dense Velocities” by Lutz & Brockmann
    - Some 25 velocity solutions provided (including the EPN densification) in well defined reference frames (preferably ETRF2000) compared and combined
    - Data cleaning is ongoing
    - More information:  
[http://pnac.swisstopo.admin.ch/divers/dens\\_vel/index.html](http://pnac.swisstopo.admin.ch/divers/dens_vel/index.html)
    - Dataset “VELF\_20180911.STA” from September 12<sup>th</sup>, 2018, is used in the following
    - Dataset is in ETRF2000



# COLLOCATION – EU DENSE VELOCITIES

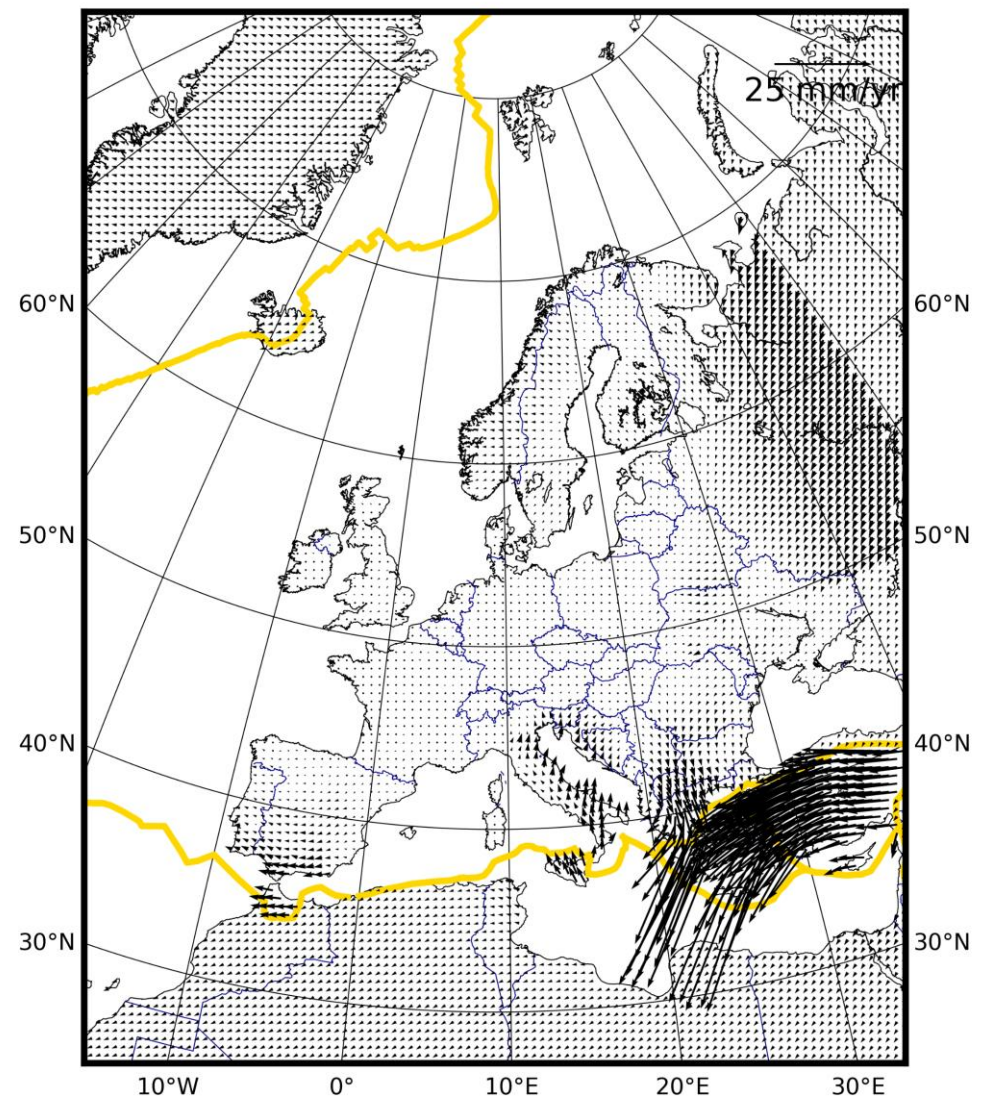


Collocation  
including plate  
boundaries

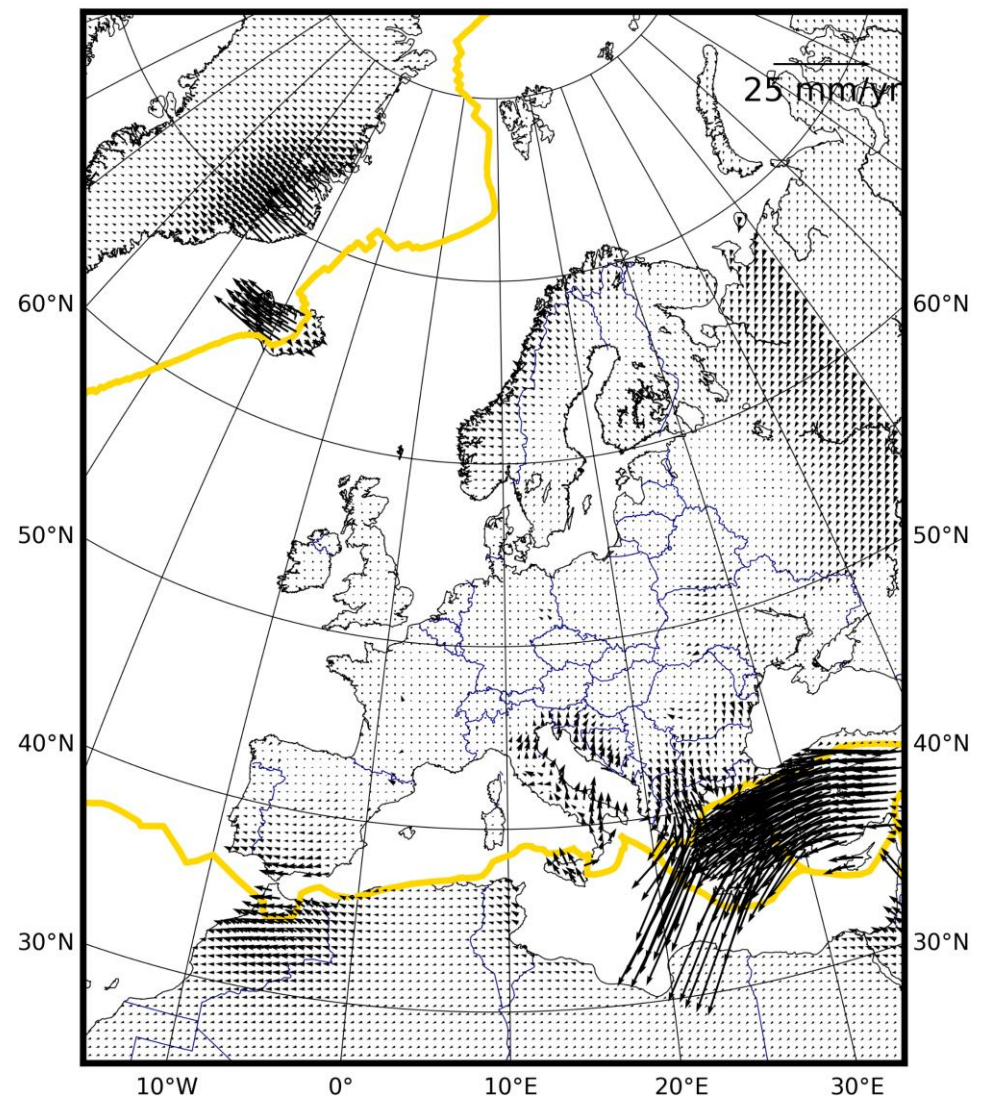


# COLLOCATION – COMPARISON

## EPN densification



## EU Dense Velocities





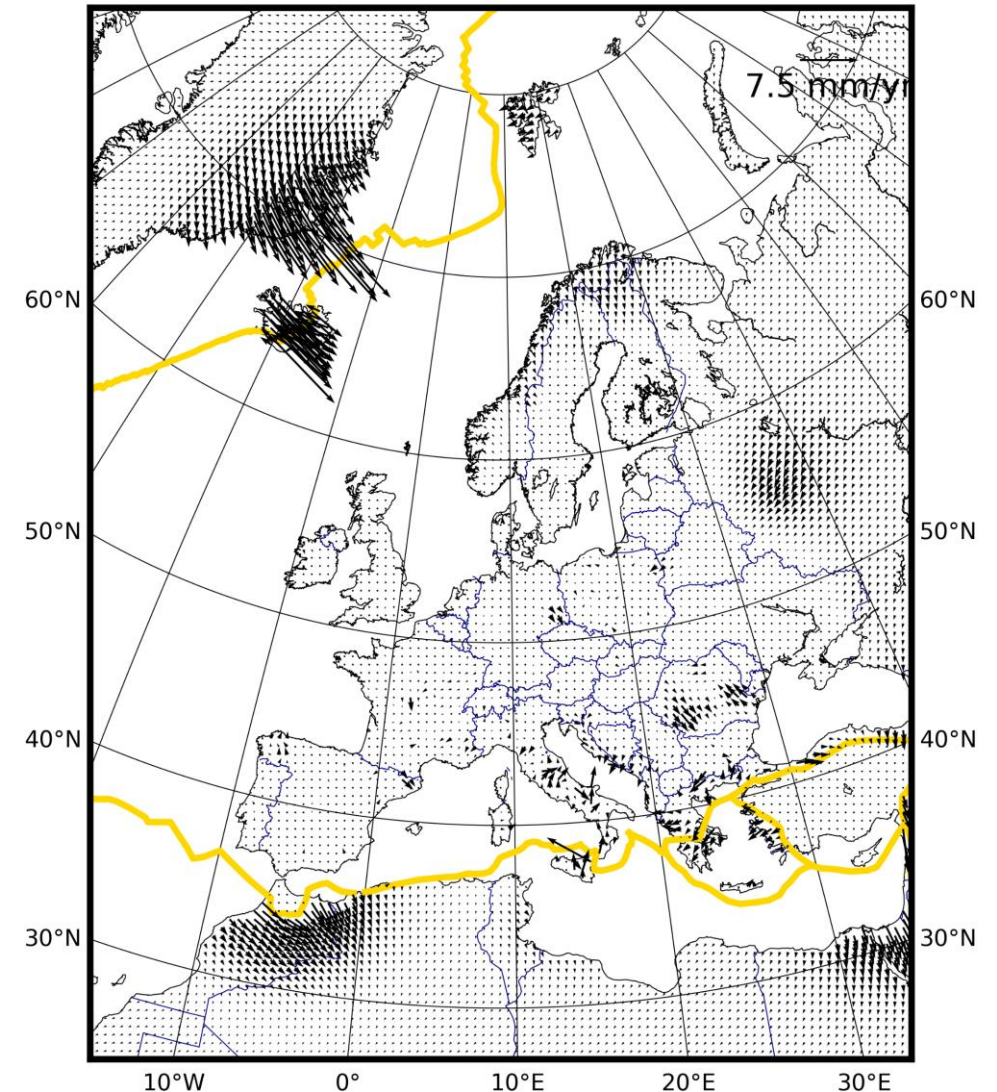
# COLLOCATION – COMPARISON

- Difference between collocated velocity fields obtained from EPN densification and EU Dense Velocities

## Entire area

	Min	Max	Mean	Std	RMS
EW	-5.386	12.660	-0.027	0.904	0.905
NS	-10.002	3.594	-0.345	0.528	0.631

## EPN densification – EU Dense Velocities



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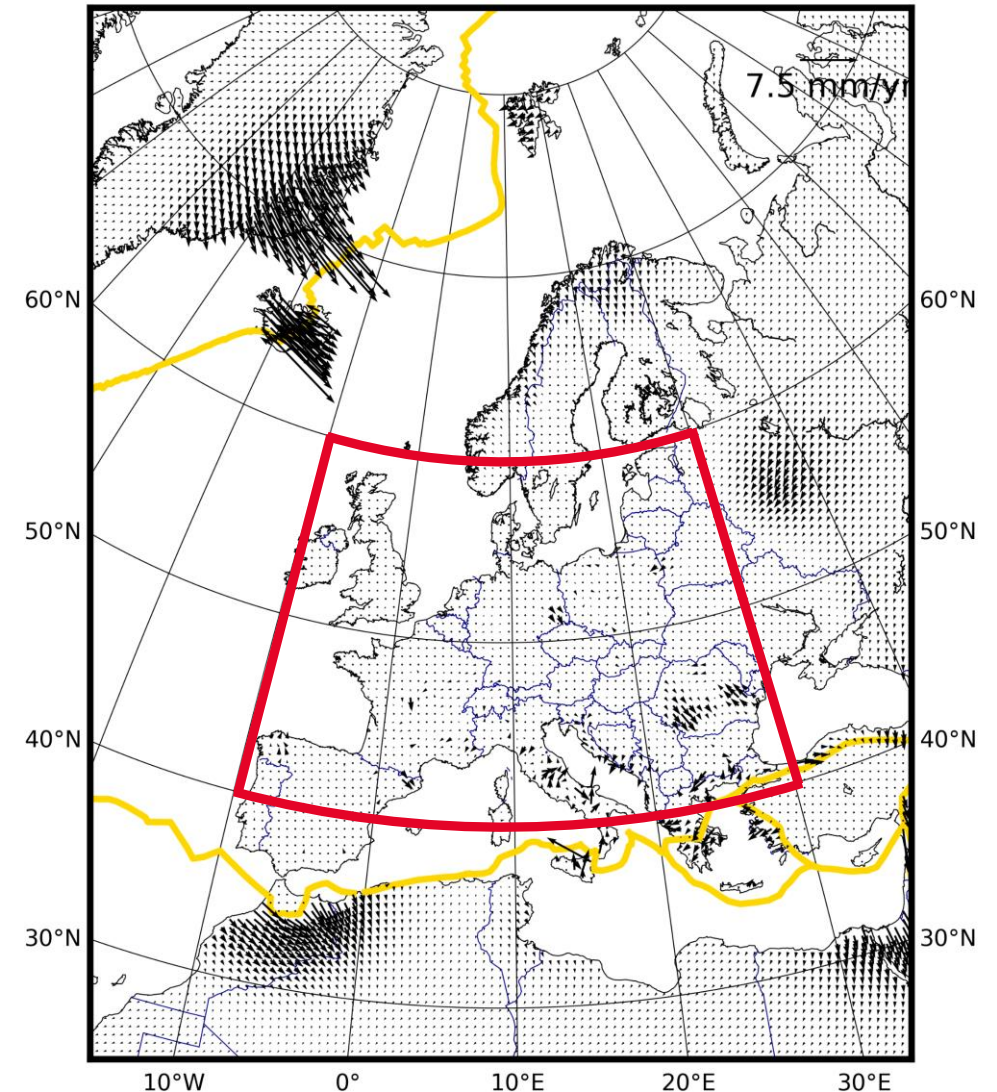
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EW	-5.386	12.660	-0.027	0.904	0.905
NS	-10.002	3.594	-0.345	0.528	0.631

## Central Europe

EW	-2.505	1.946	-0.030	0.252	0.254
NS	-2.652	2.777	-0.037	0.329	0.331

## EPN densification – EU Dense Velocities



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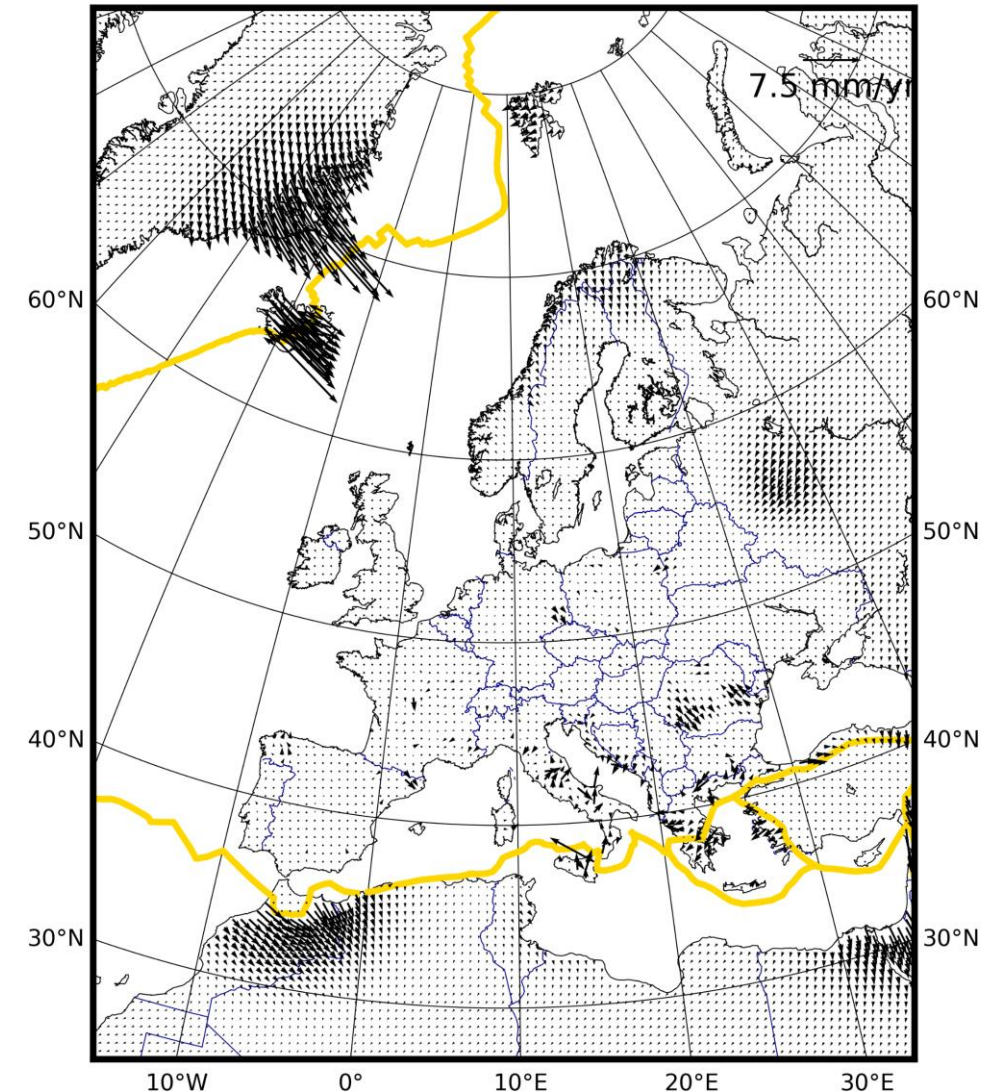
### Germany

EW	-0.583	0.698	-0.003	0.074	0.074
NS	-0.244	0.049	-0.007	0.033	0.034

### Italy

EW	-5.386	1.946	0.021	0.667	0.667
NS	-1.778	3.594	0.208	0.756	0.784

EPN densification – EU Dense Velocities



# SUMMARY

- Deformation model (velocity grid) for Europe obtained
- Collocation uses both horizontal components at the same time as well as including their correlation (follows Legrand, 2007)
- Plate boundaries implemented in collocation → provides better estimates of the horizontal velocities
- Vertical component can be also added in the collocation procedure (not shown here)
- Uncertainties can be calculated as well (formal standard error of the LSC)
- Outlook:
  - Implementing non-stationarity in covariance calculation
  - Increasing grid density as well as using high-resolution coastlines to cover all areas on land in Europe
  - Run cross-validation to obtain an external estimate of the uncertainty



THANK YOU FOR YOUR ATTENTION!

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LANTMÄTERIET

