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THE REMEASUREMENT OF THE SWEDISH NATIONAL TRIANGULATION NETWORK

2: Computations and Results

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The first systematic geodetic triangulation of Sweden dates back to the first decades of the nineteenth century. By the turn of the century a coarse-meshed system of chains of first-order triangulation had been established extending from the connection to the Danish triangulation in the south to the Russian-Swedish triangulation north of the Polar Circle, and from the Baltic coastal islands in the east to three connections to the Norwegian triangulation in the west. Within the chains a limited amount of second and lower order triangulation was carried out although the geographical distribution of triangulation stations was very irregular. Large areas of the country were without triangulation.

The purpose of this first triangulation was twofold: first to serve as a framework for the national mapping programme and secondly to provide the basis for scientific geodetic studies. Demands for an increased and more diversified mapping programme, the lack in many areas of an adequate triangulation network to which newly completed town and city triangulations could be connected and developments within international geodesy resulted in 1905 in a decision to modernise and extend the national triangulation network. A re-observation programme was prepared with the aim of establishing a first-order framework of the best achievable accuracy which included the measurement of base lines and the observation of Laplace stations. Observation of the network which comprises thirteen closed loops made up of single or double chains of triangles was begun during the first decade of the present century and completed in 1951. In addition to the angular observations, 29 Laplace stations were observed and 11 base-lines were measured.

The demands of the national mapping programme were a major fact in instigating the re-triangulation, and to keep pace with these demands the computation of the network could not be delayed until all thirteen loops had been observed. Computation of the separate loops was therefore carried out as the angular observations were completed and, thereafter, the separate loops were connected to each other. The computational problems involved in this procedure were accentuated by the fact that observation of Laplace stations and the measurement of base-lines did not keep pace with the angular observations. By 1938 five base-lines and seven Laplace stations had been completed and with the help of these the five southernmost, earlier computed, loops were subjected to a re-scaling and re-orientation. Thereafter, continuing northwards the loops were computed in nine separate stages.

External pressures had necessarily overridden purely geodetic considerations, and good quality observational data was subjected to less satisfactory adjustment procedures which led, inevitably, to deformations and scale errors in the adjusted network. The adjustment did not fulfill scientific requirements but, as has been the case in many other countries, it was considered that a distinction should be made between coordinates intended for mapping purposes and coordinates for scientific purposes and that initially the mapping requirements should be given priority. Second and third order triangulation was later carried out to fill out the areas within the first-order loops.

With the completion of the triangulation programme in 1951, the fundamental requirements of the basic mapping programme had been met, but demands for improved accuracy and increased homogeneity in the network had been steadily growing and these demands received a significant impetus with the introduction of the first E.D.M equipments and the development of new measuring techniques. Large-scale state highway surveys and similar technical developments require good quality and homogeneous systems; the importance of connecting town and city and other local surveys to the national network had long been apparent but problems arose when these networks, many of which had been established using modern equipment and methods, were to be connected to the lower quality national triangulation. The demand for high quality triangulation was an increasingly clear trend both nationally and internationally.

The question of a second modernisation of the Swedish network was, therefore, by the late 1950's a question of considerable importance.

In 1965 a Working Group was set up with instructions to investigate the need for strengthening the existing triangulation, levelling and gravimetric networks; to suggest suitable techniques for implementing such improvements as were considered necessary; and finally to present a cost framework for carrying out the recommendations.

The group reached the conclusion that the Swedish triangulation network was in need of "far-reaching improvements if the demands of those who use the triangulation are to be met and if the maximum benefit of rationalization are to be achieved".

The findings of the group were given governmental approval and in 1967 the re-observation programme was begun.

The new network

As the quality of the observational material from the 1905-1951 first-order triangulation is good, the old loops have been retained as the basic framework for the new network in which the classical layout has been densified using trilateration methods. All sides in the old chains of angles have also been measured using E.D.M methods.

By the end of 1980 when the remeasurement programme will have been completed the whole of Sweden will be covered by an area network with side lengths of approximately 30 kms. Within approximately 70% of the country this network will be broken down to an average side length of 10 kms. To reduce the need for reconnaissance to a minimum and to connect the older existing networks to the new network, the new network has been built up as far as possible on already existing triangulation stations. When completed, the Class 1 network will comprise approximately 1100 stations and Class 2 at least 2500 stations.

Over the greater part of the country connections have been made to existing town and city and other triangulation networks on a repayment basis. In addition, a considerable number of new town and city trilateration schemes have been measured.

As was the case with the first triangulation of Sweden, the new triangulation is being carried out to meet the needs of two types of consumers: geodetic scientists and land surveyors. It was clearly stated at the project study stage that whilst a geodetic scientist may be willing to wait until the whole of a network has been observed and adjusted as an entity, land surveyors will, for practical reasons, want the new coordinates as soon as possible after the field observations have been completed. This is particularly the case in Sweden since all surveys must be connected to the national network. Connections can, naturally, be made using the old coordinates but much is obviously to be gained by using the new, homogeneous network.

To, as far as possible, meet the requirement of both user categories the country has been broken down into twelve regions; fig 1. As a comparison of fig 1 with fig 2 shows, the regional boundaries follow the old chains of first-order triangulation. This "cage" of triangulation gives each region a stability in orientation which has been further strengthened by the inclusion of all older first-order azimuths supplemented by new azimuth measurements. The question of the role played by azimuths in a network of this type is discussed further in a later section.

As soon as field work is completed in a region, regional coordinates are computed and made available for use. All new observations together with the old first-order angular measurements and azimuths are included in these regional adjustments which are based on only one fixed station. To avoid problems at regional boundaries, generous overlaps are provided. In these overlap zones transformation coefficients are computed. The sizes of the regions vary considerably as fig 2 shows; and to further speed up the delivery of coordinates, Region 11 has been broken down into 11S and 11N and Region 12 into 12E and 12 W. The figures in table 6 indicate the typical size of an overlap and the figures in table 7 show the number of stations in the regions.

Field Methods

The decision to use trilateration techniques based on the use of modern E.D.M instruments constituted a radical change in first-order measuring methods. The field work involves the measurement of all the sides in the old chains as well as the 30 kms sides and the 10 kms breakdown sides in the new area network. A very limited number of angular observations have been made. To prevent errors in orientation, additional astronomical stations have been observed within the loops to supplement the older astronomical stations which lie within the chains.

The classical concept of a first-order triangulation of high accuracy followed by a breakdown to a second-order network of lower standard and with shorter triangle sides is not valid for trilateration networks, as accuracy in a 10 kms breakdown side is as good, and not infrequently better, than in 30-40 kms sides. To avoid this distinction the whole of the new network is called the Basic Network and includes two classes of sides: Class One sides of the order of 30 kms and Class Two sides of the order of 10 kms. The classification is thus a grouping based solely on side-length, accuracy requirements being the same in both classes.

Instrumentation

The range of E.D.M equipment available in 1967 was limited: laser E.D.M instruments were, for example, still at the prototype stage. Both microwave and electro-optical instruments with an accuracy which satisfied specifications were, however, available and Tellurometers and Geodimeters were already in use within the Geodetic Division. Although it was clear that the inherent precision of the Geodimeter was better than that of the Tellurometer, field experiences had shown that the Model, 2, 4 and 6 Geodimeters were, due to their dependence on good visibility and their relatively restricted range, particularly in daylight, not generally suited to large-scale trilateration programmes where time and cost factors were important. On the other hand, although the visibility factor is unimportant when using micro-wave instruments the determination of refractive index is

a complex problem if a high standard of accuracy is to be maintained. The Working Group suggested the adoption of microwave instruments as the principal instrument type with electro-optical instruments in a secondary role.

Of the available 3 cm microwave equipment the MRA 101 was chosen. To further rationalize field routines a telescopic, portable, duralumin tower was developed. The tower which is a lattice construction with a triangular cross-section weighs between 200-300 kg and can, in collapsed form, be transported on small, standard trailers or by light helicopters. Tower heights can be varied from 12 m to 18 m to 25 m with a maximum height of 30 m; and a 30 m tower can be erected by a trained party in under four hours. Forty towers are at present in use.

In connection with the measurement of the Tromsö-Catania satellite base-line in 1967-1968 a prototype laser Geodimeter was put into experimental field use in Sweden. The great potential of the laser instrument was, despite early problems, clearly demonstrated, the high degree of inherent accuracy of the standard Geodimeter being supplemented by excellent day-light range. Laser Geodimeters were slowly phased into the field programme during 1969 and on an increasing scale in 1970.

In 1971 with the introduction of modifications to the observing towers which had been specially designed for Tellurometer observations and which, initially, lacked the stability required for electro-optical measurements, laser Geodimeters replaced the micro-wave instruments as the principal instrument type. Since the 1972 field season all distance measurements have been made using laser equipment. The Geodetic Division now has six Model 8 Geodimeters in operation. Five hundred prisms are available and these can be mounted in three, seven and sixteen prism housings or in combinations of all three. Two model 6BL Geodimeters are also available for short distances.

The introduction of laser Geodimeters radically affected the organization of the field work and resulted in lowered costs and improved accuracy.

Ancillary equipment for meteorological observations is of high class and includes thermistor equipment developed at the NLS. Centering of towers is carried out using optical plummeting equipment.

The achilles heel of a trilateration programme is often the determination of station heights. Problems arose in the southern part of Sweden during the first years of the programme due to unexpected weaknesses in the quality of the existing trigonometrically determined heights. Since the early 70's new heights have been determined for all stations.

Levelling is carried out both for direct determination of station heights and in connection with height traversing. Lines of levels are run in both directions with a minimum permitted difference between the forward and reverse height differences of $15\sqrt{D}$ mm, where D is the distance in km.

Height traversing is carried out where, due to difficult terrain conditions, direct levelling is not possible. Maximum permitted traverse length is of the order of 3000 m. Normal procedure is to run levels as near as possible to the triangulation station and to make the final connection by traversing. Two separate determinations of the height difference are made from two independent instrument set-ups. The maximum permitted difference between the two height traverse determinations is 50 mm. The model 700 Geodimeter is particularly suitable for height traversing. A minimum of two sets of vertical angles are observed together with a single determination of the distance.

Reduction to centre measurements are carried out with great care using good standard, calibrated equipment. Spring balances are used to stretch the tape with the correct tension. Air temperatures are recorded for reduction purposes. To avoid taping errors all distances are measured twice using different zeroes on the tape. Height differences for distance reductions are determined by levelling or from vertical angles. Directions are observed using second-order theodolites. At least two known stations must be observed for orientation purposes. Where theodolite observations for orientation purposes are not possible, gyro theodolites are used. For eccentricities of up to 20 m standard observing procedures are used. For longer distances special observing procedures are required.

Directions and distances to at least two witness marks are also measured. As many redundancies as possible are sought. To provide extra redundancies temporary pegs are sometimes set out. Distances over 100 m are measured using Geodimeters. All measurements are computed in the field using small, portable electronic calculators.

Field astronomy has been carried out by specialist groups. For receiving time signals one astronomy group is equipped with an Eddystone E.C. 10 S.W. receiver and the other group with a Heathkit Mohican S.W. receiver. An Eddystone 850/4 L.W. receiver is also available.

Sidereal time is recorded using three main types of chronometer: a Golay electronic quartz-crystal chronometer, an electric Le Roy chronometer and clockwork driven Nardin Marine Chronometers. One of the marine chronometers, which is still in use, was used by the explorer Sven Hedin during his journeys in Central Asia before it was handed over to the Geographical Survey (now the National Land Survey).

Time events are registered using two Favag M427 recording chronographs. These are equipped with two pens, one for recording the time trace from the chronometer and the other for recording events.

An Elsec Chronocord type 680 was purchased in 1969 but this equipment has only been used to a limited extent in the main programme due mainly to problems with the printer unit. The 680 model was replaced by a 681 for the 1973 field season. This model has functioned satisfactorily.

Careful and regular calibration of all equipment is a prerequisite for any first-order programme. The quality of the final results is as much dependent on carefully applied calibration routines as on fixed observing routines.

Frequency checks on the EDM instruments are regularly carried out during the field season. Zero constants are determined for the Geodimeters before and after the field season; for the Tellurometers, cyclic errors were also determined and an extra mid-season calibration was carried out in addition to the pre and post season calibrations. Meteorological equipment is also carefully calibrated. Thermometers are calibrated once a year. A lowtemperature chamber is available at NLS for calibration at temperatures below 0° C.

Assman psychrometers are not designed for the type of observations that are made in connection with distance measurements where almost continual temperature recordings are made for three or four hours each day during a six months period. The speed of the clockwork driven fan must, therefore, be regularly checked in the field. This is a relatively simple but nevertheless important check.

The Baromec equipment is calibrated before and after the field season against mercury standards. During the field season all instruments in each group are all compared with each other at least once each day. In addition, the field instruments are checked at monthly intervals against three "master" Baromecs which are taken out into the field from the office. The "master" instruments are calibrated before and after the comparison against an Hg standard.

Computer Programmes

A comprehensive programme library has been built-up to meet the requirements of the remeasurement programme. The programme block is made up of a series of modules such that the output from one module can be used as input for the following module. Punched cards and data files are used as the input medium. The system is not integrated in the full sense of the word such that the complete computation of a network is made in one run of the computer. Due to the very

large data volume and the associated risk of introducing errors NLS has preferred to carry out the computation in separate steps, each step being carried out in a programme module.

The following programmes are at present in use :

Barkal

The programme is used to compute Baromec corrections from the large number of field comparison observations and absolute calibrations.

The results are stored on data files, together with thermometer calibration tables, Geodimeter calibration tables, Tellurometer and Geodimeter instrument constants, Tellurometer and Geodimeter frequency tables. Each file is defined by year, instrument and type of calibration, and is accessible during computer run-time.

Programme 903

This programme is used for:

- 1 the reduction to centre computations
- 2 computing (t-T) corrections
- 3 computing distance measurements
- 4 computing coordinates for excentric points - witness marks etc -
- 5 computing gyro-theodolite observations

Input is the raw field data and the approximate coordinates for the triangulation stations. The coordinates for new stations are normally computed using programmable desk-top electronic calculators. Otherwise the available older coordinates are used. This manual routine has proved to be of great value from a data-cleaning point of view.

The computations are carried out in the Gauss-Krüger projection system with a full choice of reference ellipsoid. The number of stations for which excentric coordinates can be computed is 250 with a maximum of nine excentric points at each station.

Measured directions are reduced to the plane and to the central mark if they have been observed from a satellite set-up. The finally reduced directions are checked against the bearings computed from the approximate coordinates to locate gross errors. A-priori standard errors are allocated to each separate direction. Measured distances are fully reduced during run-time. All necessary corrections are computed and these include, refraction corrections which are computed using field observations of temperature and pressure corrected using calibration data stored in the

data files, first and second velocity corrections, frequency corrections, instrument zero and cyclic corrections, geometrical corrections, reduction to centre corrections and projection corrections. As a check, the finally reduced projection distance is compared with the distance computed from the approximate coordinates. A priori standard errors are computed for each separate measurement as well as for the mean of a set of measurements.

The results which, normally, will be used as input for the adjustment stage can be stored either on data files or on punched cards. A lineprinter print-out is always available. Errors can be erased and corrected on the data files using simple routines. New data can also be added to existing files.

The programme includes a large number of error-search routines and feasibility checks with associated print-outs. The out-data lists include:

- 1 Station number, name, X, Y and Z coordinates for the central mark and all excentric points together with distance and bearing from the central marks to each excentric point.
- 2 Reduced directions with a print-out of (t-T) corrections, satellite reduction corrections, à priori standard error and the residual (reduced direction minus direction computed from coordinates).
- 3 Reduced distances with a print out of the input data, computed value for refractive index, slope distance, elliptic distance, projection distance and the projection distance reduced to the central mark. For each distance an à priori standard error is given. The mean of a set of distances is compared with the distance computed from coordinates and an à priori standard error is computed for the mean of the set.

For Geodimeter measurements the computed value of each frequency is also given. For Tellurometer measurements the theoretical refractive gradient is compared with the measured values. If required, a print-out of the calibration tables used for the computations can be made.

The weighting of observations

The weighting of combined observations has been the subject of a series of statistical studies at NLS. The general principles adopted in 1967 for distance and direction measurements have been the subject of only minor modifications and are still applied at the present time. The weighting of azimuth observations has been considerably changed.

A priori standard errors are either fed into the computer together with the raw data as in the case of direction observations or in tabular form from which standard errors are computed during run time for distance measurements. It may be of interest to discuss weighting in more detail.

As a general rule, in southern and central Sweden, directions were observed to signal lamps on at least three nights with a direction weight of 18 respectively 20, 21, 24. The Schreiber observing routine was normally used. Directions measured in this way have been allocated a standard error of $1,0^{\text{CC}}$. For measurements carried out on only one night the standard error is increased $\times \sqrt{2}$. Where the observing programme has a lower direction weight than 18 (respectively 20, 21, 24) the à priori standard error is changed to $\sqrt{18}$: actual direction weight. This has been the case, for example, where additional observations have been made with a lower direction weight than the original observations.

In northern Sweden (north of $62,6^{\circ}\text{N}$) observations were often made on two consecutive nights. Statistical studies of these observations have revealed systematic errors. The à priori standard error of these observations has been set at $1,3^{\text{CC}}$. For observations made on only one night this standard error is multiplied by $\sqrt{2}$. All the older angular observations have been carefully re-checked and a number of errors were located at this stage.

An observed azimuth \mathcal{L}^* is reduced to the geodetic azimuth \mathcal{L}_g as follows

$$\mathcal{L}_g = \mathcal{L}^* - (\lambda^* - \lambda) \sin \varnothing$$

where

λ^* = longitude determined by astro

λ = geodetic longitude

The observational error in \mathcal{L}_g is

$$\delta \mathcal{L}_g = \delta \mathcal{L}^* - \delta \lambda^* \sin \varnothing$$

since λ can be considered to error-free.

λ^* is normally determined as a difference between the reference longitude station and the field station. In the ED 50 adjustment, azimuths were considered to be "error free". In the adjustment of the Tromsö-Catania satellite baseline traverse the allocated à priori standard error was $2,0^{\text{CC}}$. The computed corrections to the azimuths were, however, significantly larger than the à priori standard errors by up to $\times 3$. In a statistical study of all the observational data - azimuths, directions and distances - both Bartlett and F-tests revealed variances for azimuths which differ significantly from variances for the other directions and distances.

Further studies of azimuth material have indicated the presence of significant systematic errors.

In the new network, the à priori standard errors for azimuths has been set to between 2,2 - 3,5^{cc} and have been computed as follows:

$$m^2 = m_{\alpha}^2 + (5 + m_{\lambda}^2) \sin^2 \varnothing$$

Where m_{α} = standard error for an azimuth dermination
 m_{λ} = standard error in the longitude

The à priori of a measured distance is computed for a Tellurometer measurement from

$$m = a + b \cdot D \cdot 10^{-4} \text{ cm}$$

and for Geodimeter measurements from

$$m = a + b \cdot D \cdot 0,7 \cdot 10^{-4} \text{ cm}$$

where

a and b are coefficients

D is the distance in m

The standard error of the mean of a series of measurements is

$$\bar{m} = 1 : \left(\frac{1}{m_1^2} + \frac{1}{m_2^2} \dots \right)$$

The coefficients used for computation of the à priori standard errors for measured distances are given in table 1.

Programme TRUT

The programme was originally written for an IBM 7044 computer, but has been expanded and revised for use on a CDC 6600 with a core storage of 100 K and a word length of 60 bits.

The adjustment method on which the programme is based is the variation of coordinates method.. Computations can be carried out on the plane or on any chosen ellipsoid. Five versions of the programme are available for a wide range of adjustment problems. The maximum number of unknowns which can be treated in a single run is 330 ie 165 new stations. The programme has been adapted to the Helmert block method and has an automatic connecting procedure.

The following data is input:

- 1 Coordinates of fixed points
- 2 Approximate coordinates for new stations

- 3 Observed direction series with à priori standard errors
- 4 Measured distances with à priori standard errors
- 5 Base-lines
- 6 Laplace azimuths with à priori standard errors
- 7 Correlated observations
- 8 Contributions to normal equations
- 9 "Free" and fixed scale factors

Input can be on punch cards, punch cards from 903 output or 903 data files.

The computations are carried out with double precision iteratively. The number of iterations can either be chosen by the computer or will continue until the value

$$\frac{\sum p_{11} - \sum p_{v\bar{v}}}{\sum p_{11}} = < 10^{-6}$$

Output includes:

- 1 Coordinates for the fixed stations
- 2 Coordinates for new stations
- 3 Lists of corrected bearings with à priori standard errors, corrections to observed values and orientation quantity
- 4 Lists of corrected distances with à priori standard errors and corrections to observed values
- 5 Lists of corrected azimuths with à priori standard errors and corrections to the observed values
- 6 In tabular form, station for station, corrections to the approximate coordinates together with standard errors in the coordinates for each new station relative both to the fixed stations (Swedish: regional standard error) and relative to those new stations with which it has contact through angular or distance measurements (Swedish: local standard error). The table also includes the standard error of unit weight and the number of redundancies.
- 7 In tabular form, information regarding: the total number of series of directions observed at each station, the total number of directions observed to and from each station, the total number of azimuths and distances measured to and from each station, the total number of measurement (directions + azimuths + distances) and the $\sum p_{v\bar{v}}$ for each measurement type.

- 8 If requested, standard errors along and perpendicular to any line joining two stations in a block (relative line errors) can be obtained.

The print-out also includes a number of intermediate checks such as comparisons between measured quantities and the equivalent values obtained from the approximate coordinates, and error information. A print-out of standardized observation equations can also be requested; and in connection with non-definite adjustments the buffer matrix can also be printed out.

Programme Block

The programme is used, in conjunction with the Helmert method for the adjustment of large networks. The network is automatically broken down into blocks based on given block boundaries and data files are made for input to TRUT. Block boundaries must form a closed polygon with a maximum of 10 corner points. The programme can only be used with plane coordinates. The maximum number of stations is 2000. A total of 10 000 series of observations including directions, distances, azimuths can be handled. The total number of blocks permitted is ten, with a maximum of 330 variables in each block. The variables comprise inner and outer unknowns, inner and outer scale factors, inner and outer azimuth unknowns and orientation unknowns. Block is used as an interface between 903 and TRUT using in-data from the former and producing out-data for the latter. The program has eliminated the need for laborious, manual, error-prone methods.

The Computations

At the time of writing, eleven of the twelve regions have been observed and computed and sub-region 12E which has been observed during 1979 will be computed during the present winter. Coordinates are available for 3250 stations. As was stated above, each region has been computed separately based on a single fixed station. In the first nine computed regions, the coordinates for the fixed station were either in the old national system (2,5^g W 1938) or in the "Map Coordinate" system. The "Map Coordinate" system was a recomputation of the old primary network as a single block and is, therefore, an updated 2:5^g W 1938 system. In the last two regions to be computed the fixed stations have been chosen in the overlap zone whereby coordinates obtained in the new adjustment have been used.

Increased computer capacity has resulted in modification of adjustment routines. In Regions 1-4, 6 and 7, due to capacity limitations, it was necessary to compute the Class 1 network first and thereafter the Class 2 network with the Class 1 stations as fixed.

In the remaining regions, increased computer capacity has made it possible to adjust both Class 1 and Class 2 stations simultaneously. These single-run adjustments, with a large number of redundancies, give a more homogeneous result than the two-stage adjustments where the Class 2 network is subjected to constraint by a Class 1 network which suffers from the lack of the additional strength that a simultaneous adjustment of all stations would have provided. To correct this type of deficiency Region 2 has been recomputed.

In table 2 the ratio between the number of observations used and the number of stations computed is given for 4 regions. The figures reveal the large volume of data which is included in an adjustment of this size and are typical for the network as a whole.

Before the final adjustment is made, a series of smaller adjustments of the new measurements are run to finally clean the data. Normally, the region is broken down into blocks of between 15 and 20 stations which are then adjusted with one fixed station and a fictitious azimuth across the block. After error search and correction, two or more of the blocks are put together and a new adjustment is made. The error-search procedure is repeated until the block size reaches a value close to the maximum TRUT capacity which is permitted without applying Helmbert principles. The coordinates obtained in this manner are then compared with those used in the 903 run from which the distances used in the test computations were obtained. If significant changes in the coordinates are revealed, 903 is re-run.

To maintain the strength of the outer edges of the regions one or two rows of stations outside the boundary are included in the adjustment. The coordinates for these stations are, however, excluded from the lists of final coordinates. This procedure ensures that the outermost stations in a region are not computed using a "lop-sided" distribution of measurements.

After the final adjustment has been made and approved, programme 903 is re-run once more, using the definite coordinates, to obtain definite coordinates for all excentric stations. In this run no reduction of the field observations is made.

The Results

In the eleven, completed, regional adjustments made so far the standard error of unit weight has varied between 0,7 and 1,3. Although this figure, which is the relationship $\frac{\sigma_{\text{à posteriori}}}{\sigma_{\text{à priori}}}$, is in no way indicative of the

quality of the network without knowledge of the allotted

à priori standard errors (see above), it does clearly indicate that these values were correctly chosen. As the chosen à priori standard errors meet international requirements for primary networks a value close to unity should be indicative of a good quality network.

Average corrections to measured distances are of the order of 1: 500 000, and only in a few cases do corrections to observed directions exceed the allocated à priori standard error. In those few cases where angles and distances do not fit, examination of the "Map Coordinate Adjustment", which was basically a classical adjustment with relatively few measured distances, reveals similar tendencies which would confirm weaknesses in the directions and not undue constraint from the distance measurements.

The problem with azimuths is apparent in all regions. Special studies have clearly shown that the relatively limited number of azimuths tend to be drowned by the great volume of direction and distance measurements and they have a very restricted influence on the orientation of the network. It is, therefore, difficult to know whether the corrections obtained, although they seldom exceed the relatively large à priori standard errors, are the result of errors inherent in the azimuths themselves or the result of external constraint on them. It would be interesting to carry out a study of the interaction of directions, distances and azimuths where the balance between them is better than in Sweden.

In table 3 mean and maximum local and regional standard errors in coordinates in 4 regions are given. The local standard errors are the most interesting as they indicated how well breakdown networks can be expected to fit. The size of the regional standard errors varies according to the placing of the fixed station, i.e. whether it is in the middle or corner of the block. A clear relationship exists between the value σ_x and σ_y for a station and its location relative to the fixed station. The relationship indicates a very stable scale and a less stable orientation. In table 4, a number of standard errors along and across long lines in four regions are given. These are definitely indicative of the high quality of the networks. The value s is the standard error of the distance, $s \cdot AZ$ is value obtained in cm by multiplying the distance by the standard error in azimuth, and $R/AVST$ is equal to

$$\sqrt{(s)^2 + (s \cdot AZ)^2} : \text{distance}$$

The results of Helmert transformations computed using common stations in five separate overlap zones are given in table 5. The large number of common stations is indicative of the generous size of the overlaps. see table 6.

The results reveal the generally high degree of homogeneity in the new network and clearly show the even better quality of the regions in north-central and northern Sweden where only Geodimeters have been used. In Regions 1-7 and 9 the number of Tellurometer measurements is greater than the number of Geodimeter measurements, whereas in Regions 8,10,11 and 12E only Geodimeters have been used. In addition to being of a less homogeneous quality, Tellurometer measured distances are systematically shorter than Geodimeter measurements. In "Geodimeter" regions where only a very limited number of Tellurometer measurements have been made the Tellurometer distances have been re-scaled to the Geodimeter scale, during the final adjustment. As was stated above, improved adjustments techniques have also significantly improved net homogeneity.

A series of doppler measurements have been made between stations in the new network. At the time of writing only the distance Mårtsbo - Hovfjället is available. The values should be seen as provisional. A comparison between the distance computed from regional coordinates and the doppler distance is given in table 8.

The true test of any network is the results which can be obtained in breakdown networks. During the course of the remeasurement programme a large number of town and city networks have been measured. These have, without exception, fitted without any problems.

When Region 12W has been measured and computed the whole of Sweden, 450 000 km², will be covered by a network of high quality. The final stage of the programme, a scientific adjustment of the whole country, will then be carried out.

Conclusion

The remeasurement of a country is necessarily a question of teamwork involving co-operation between groups of planners, field teams, computers, programmers and many other persons. But there are individuals who always rise above the collective and leave an indelible mark on the pattern of events. One such person was Ilmar Ussisoo. Ilmar who died suddenly whilst this paper was in the course of preparation, has contributed more than any other single person to the launching and successful carrying out of the remeasurement programme. With his death, the National Land Survey of Sweden and international geodesy has lost a gifted geodesist and a true colleague.

References

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Table 1.

Computation of standard errors for distance measurements

TYPE OF INSTRUMENT	k	a	b	α
Geodimeter 6BL	0.20	1.0	2.1	
Geodimeter 4, 6	0.20	1.5	2.1	
Geodimeter 8	0.20	1.0	2.1	
Tellurometer MRA 2	0.25	6.0	3.0	7.2
Tellurometer MRA101	0.25	3.0	3.0	72.0
Tellurometer MRA 4	0.25	2.0	3.0	72.0

k is used for computing 2nd velocity corrections

α is used for computing cyclic error corrections

a is a constant = Σ zero error + set-up error +

b is a distance dependent correction

Table 2.

Ratio. between the number of observations and the number of new stations

Region	Number of obs=a	Number of stat.=b	a/b	stand. error of unit weight
2	1064	252	4.2	0.84
8	376	61	6.2	0.97
9	1601	400	4.0	0.80
11S	3165	522	6.1	0.70

Table 3

Mean and maximum local and regional radial standard error in coordinates

Region	σ local rad		σ regional rad	
	max	mean	max	mean
2	7.5	1.9	9.0	3.1
8	14.1	1.8	34.0	14.5
9	6.9	2.2	26.9	5.2
11S	11.8	1.7	38.3	5.6

Table 4

Standard errors of long lines across regions

SIDE	DISTANCE m	σ S cm	σ S * AZ cm	σ R/A (cm/km)
<u>REGION_5</u>				
4380 - 71740	217639.413	18.7	43.9	0.22
5700 - 64530	211873.908	16.0	34.8	0.18
63990 - 5700	196894.628	14.3	32.9	0.18
<u>REGION_7</u>				
5451 - 96221	67799.822	3.6	3.8	0.08
15770 - 25960	60432.493	7.1	11.7	0.23
94530 - 85900	47854.004	4.8	8.6	0.20
<u>REGION_9</u>				
28290 - 25530	180833.618	4.9	27.2	0.15
28290 - 29740	36260.888	2.3	6.1	0.18

Table 5.

Result of the computation of transformation coefficients in the overlap zone between regions.

FROM REG-TO REG	SCALE FACTOR	σ ORIENTATION	CORRECTION TO COMMON STATION m.		
			0.00 ^m -0.050	0.051 ^m -0.100	0.101 ^m -0.163
7 → 9	1.000000671	- 4.08 ^{cc}	52%	46%	2%
6 → 7	0.999998963	+ 2.12 ^{cc}	85%	15%	-
5 → 7	1.000000287	- 0.62 ^{cc}	15%	49%	36%
10 → 11	0.999999958	- 0.19 ^{cc}	100%	-	-
2 → 3	0.999999135	+ 4.08 ^{cc}	87%	12%	1%

Table 6.

Computation of transformation coefficients. Comparison of the results

FROM REG-TO REG	m _x . m	m _y . m	μ . m	NUMBER OF COMMON STATIONS
7 → 9	0.034	0.021	0.040	213
6 → 7	0.020	0.014	0.025	102
5 → 7	0.037	0.061	0.072	99
10 → 11	0.007	0.004	0.008	127
2 → 3	0.020	0.016	0.025	75

$$m_x = \sqrt{(v_x v_x):r}$$

$$m_y = \sqrt{(v_y v_y):r}$$

$$\mu = \sqrt{m_x^2 + m_y^2}$$

r = number of redundancies

Table 7

Number of stations in each region. Stations which fall within the overlap are included in two or more regions

Region	Class 1	Class 2	Σ
1	68	138	206
2	68	184	252
3	54	171	225
4	58	89	147
5	107	444	551
6	136	492	628
7	81	415	496
8	19	42	61
9	81	319	400
10	155	545	700
11	324	480	804
12	93	150	243

Table 8

Comparison between the doppler distance and the distance computed from coordinates between Mårtsbo and Hovfjället

Doppler m	Coordinates m	Δ	prop Δ
238607.58	238607.73	0.15	1:1600000

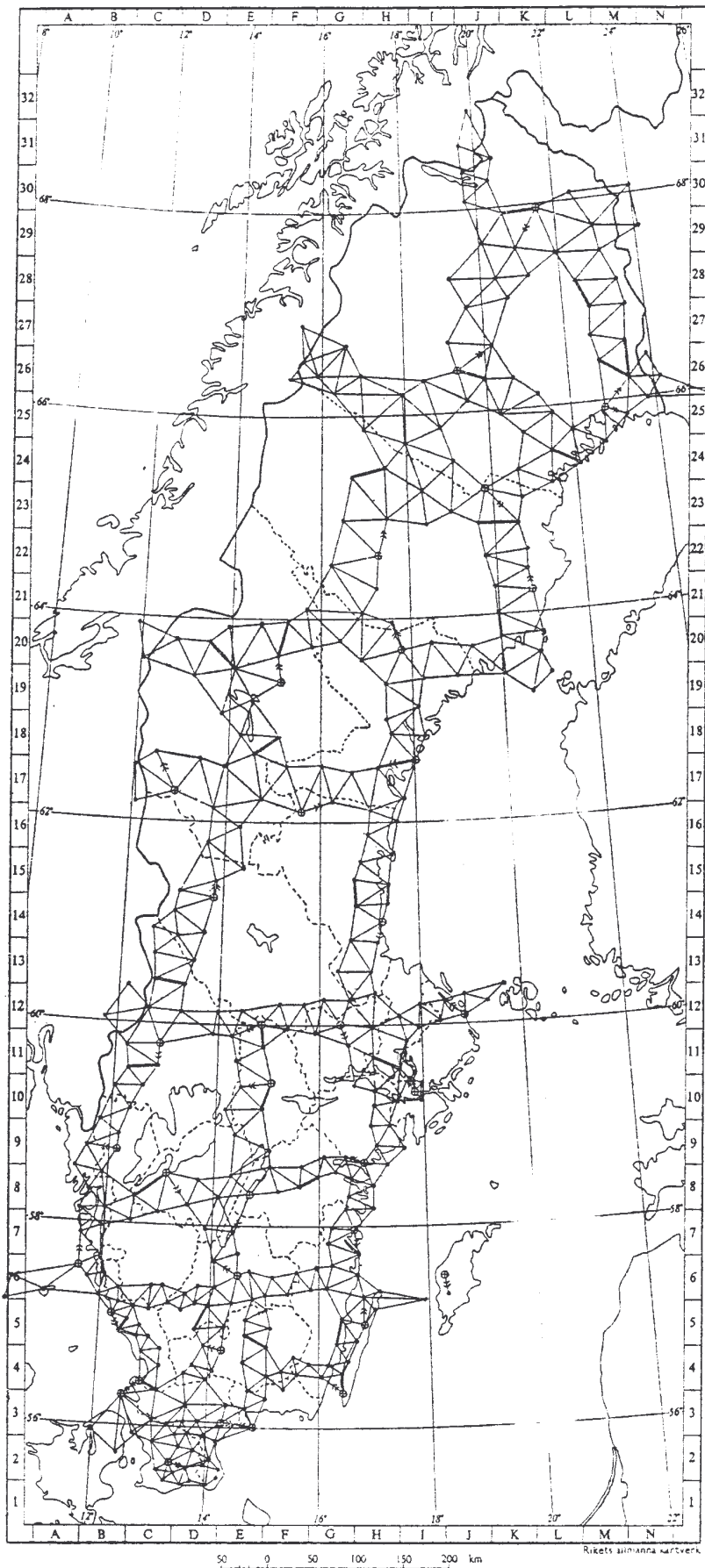


Fig. 1

The Swedish First-Order
Triangulation 1905-51

THE NATIONAL TRILATERATION NETWORK

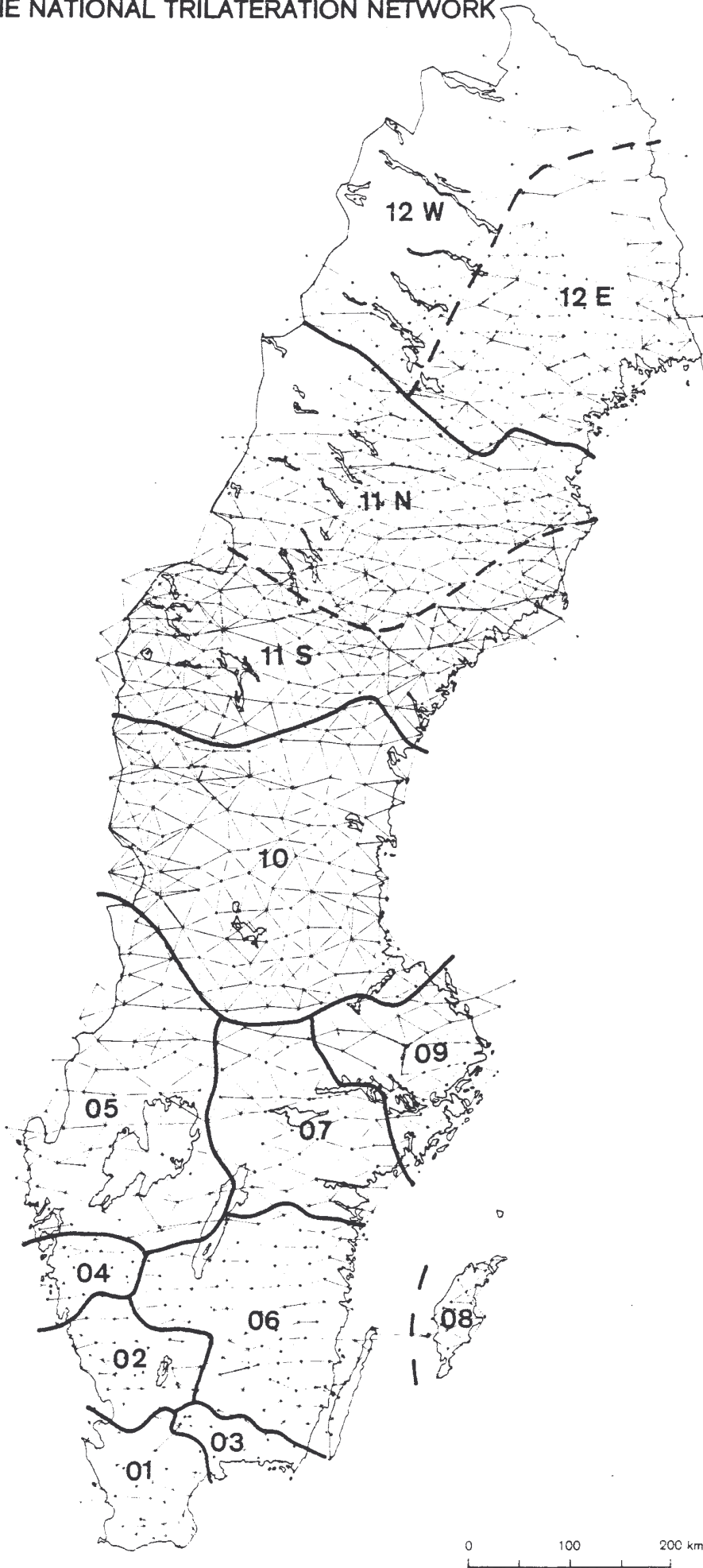


Figure 2.