Transformation between ITRF 2014/WGS 84 and SWEREF 99

Introduction

This memo describes how to transform coordinates in the latest realisation of the International Terrestrial Reference System (ITRS), the International Terrestrial Reference Frame 2014 (ITRF 2014), to the national reference frame of Sweden (SWEREF 99). Such a transformation is, for example, needed for transforming the coordinates obtained from a global PPP (Precise Point Positioning) service, which usually gives the coordinates in ITRF (or WGS 84) at the current epoch, to SWEREF 99.

The World Geodetic System 84 (WGS 84) is the reference system used by GPS. Thus, coordinates obtained by standard GPS positioning (without connecting to any reference stations) will be in this reference system. New realizations of WGS 84 are made from time to time, and the latest realizations have been connected to the ITRF at that time. WGS 84 will agree with ITRF on the centimetre level, and the most accurate way to realize WGS 84 is by ITRF. Hence, users who need to transform between WGS 84 and SWEREF 99 should use the transformation described in this memo.

The ITRS is a dynamic reference system, meaning that it contains both positions and velocities. The reason is that on a global scale one needs to consider the continental drift, which can be several cm per year. However, in SWEREF 99 (as well as all other national realisations of the European reference system ETRS 89) the Eurasian plate is held fixed to the position it had at the epoch 1989.0, thus the continental drift can be ignored. Still it is necessary to consider crustal deformations, like those caused by postglacial land uplift. This effect can reach up to 1 cm per year vertically and a few millimetres per year horizontally. In SWEREF 99, this is handled by applying a model of the effect, calculating all coordinates back to the reference epoch 1999.5.

Hence, to perform the transformation we need to consider both the motion of the Eurasian plate and the crustal deformations in the Fennoscandian area. The former can be handled by using the plate model produced from ITRF 2014, while for the latter a velocity model describing the deformations is needed. In the process producing the transformations between ITRF 2014 and the national reference systems of the Nordic and Baltic countries (like SWEREF 99), the NKG (Nordic Geodetic Commission) produced a new velocity model, NKG_RF17vel. This model is based on theoretical predictions of a GIA (Glacial Isostatic Adjustment) model plus empirical corrections based on GNSS-derived velocities in the Nordic/Baltic area.

Transformation ITRF 2014 to **SWEREF 99**

The transformation described here transforms coordinates in ITRF 2014 at some epoch t_c to SWEREF 99. The transformation consists of four steps.

1. Transformation from ITRF 2014 to ETRF 2014

In the first step the coordinates are transformed to the European Terrestrial Reference Frame 2014 (ETRF 2014), which is a realisation of ETRS 89 based on with ITRF 2014. This is done by rotating the Eurasian plate back to its position in 1989. Given the rotation velocities $\dot{R}_X$, $\dot{R}_Y$, and $\dot{R}_Z$, this transformation is:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{t_c}^{ETRF2014} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{t_c}^{ITRF2014} + (t_c - 1989) \begin{bmatrix} 0 & -\dot{R}_z & \dot{R}_y \\ \dot{R}_z & 0 & -\dot{R}_x \\ -\dot{R}_y & \dot{R}_x & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{t_c}^{ITRF2014} \quad (1)$$

The rotation velocities are given in Table 1.

Table 1: The rotation velocities needed for equation (1).

$\dot{R}_X$ [mas/year]	$\dot{R}_Y$ [mas/year]	$\dot{R}_Z$ [mas/year]
0.085	0.531	-0.770

Note that these values (given in mas/year) need to be converted to rad/year by multiplying with the factor $\pi/(180 \cdot 3600 \cdot 1000)$ before inserted into Eq. (1).

2. Change epoch to 2000.0 by correcting for crustal deformations

The second step is to change the epoch to the intermediate epoch 2000.0. This is a common epoch chosen within the NKG as a common intermediate epoch to be used when doing the transformation between ITRF 2014 and the national reference frames of the Nordic and Baltic countries. The change of epoch is done by correcting for the land uplift, both vertically and horizontally, between 2000.0 and the epoch t_c. To do this, the NKG_RF17vel model for the intra-plate deformations in the Nordic and Baltic region was developed within the NKG. This model gives the velocities of the deformation provided on a grid. These are stored in three different grid-files, one for the North, East and Up component, respectively. The grid-files are available in different formats, e.g the gri-format (files NKG_RF17vel_n.gri, NKG_RF17vel_e.gri, and NKG_RF17vel_u.gri). The velocity to be used for the transformation is obtained through interpolating the grids, a simple bi-linear interpolation is good enough. This gives the velocity in North, East, and Up directions. This needs to be converted to cartesian coordinates by:

$$
\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{NKG\_RF17vel} = \begin{bmatrix} -\sin\varphi\cos\lambda & -\sin\lambda & \cos\varphi\cos\lambda \\ -\sin\varphi\sin\lambda & \cos\lambda & \cos\varphi\sin\lambda \\ \cos\varphi & 0 & \sin\varphi \end{bmatrix} \begin{bmatrix} V_n \\ V_e \\ V_u \end{bmatrix}_{NKG\_RF17vel} \tag{2}
$$

Here φ and λ denotes latitude and longitude, respectively. Then the change of epoch to 2000.0 is calculated by:

$$
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{2000}^{ETRF2014} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{t_c}^{ETRF2014} + (2000.0 - t_c) \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{NKG\_RF17vel} \tag{3}
$$

3. Apply a seven parameter Helmert transformation

The next step is to apply a seven parameter Helmert transformation to account for the difference between ETRF 2014 and SWEREF 99 at epoch 2000.0. This is performed by the equation:

$$
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{2000.0}^{SWEREF99} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} + (1+D) \begin{bmatrix} 1 & -R_z & R_y \\ R_z & 1 & -R_x \\ -R_y & R_x & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{2000.0}^{ETRF2014} \tag{4}
$$

The parameters needed for this transformation were determined within the NKG, and are given in Table 2.

Table 2: Transformation parameters for transforming between ETRF 2014 to SWEREF 99 (epoch 2000.0), needed for equation (4).

T_x [mm]	T_y [mm]	T_z [mm]	D [ppb]	R_x [mas]	R_y [mas]	R_z [mas]
30.54	46.06	-79.44	3.002	1.41958	0.15132	1.50337

Note that also here values of the rotations (given in mas) need to be converted to rad by multiplying with the factor $\pi/(180 \cdot 3600 \cdot 1000)$.

4. Change the epoch to 1999.5

The final step is to change the epoch to 1999.5, the reference epoch of SWEREF 99. This is again done by using the velocities obtained in step 2:

$$
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{1999.5}^{SWEREF99} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{2000.0}^{SWEREF99} + (1999.5 - 2000.0) \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{NKG\_RF17vel} \tag{5}
$$

Thus, the final SWEREF 99 coordinates are obtained.

It should be noted that since the difference between the ITRF 2014 and SWEREF 99 is small (less than a meter) the transformations could be simplified without any significant loss of accuracy. For example, it makes no significant difference in what order the four steps are executed. Thus, in the second step the epoch could already be changed directly to 1999.5, thus

reducing the number of steps to three. Furthermore steps 1 and 3 could be combined.

Transformation SWEREF 99 to ITRF 2014

To transform coordinates from SWEREF 99 to ITRF 2014 the above described transformation needs to be run backwards. Thus, first change the epoch to 2000.0:

$$
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{2000.0}^{SWEREF99} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{1999.5}^{SWEREF99} + (2000.0 - 1999.5) \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{NKG\_RF17vel} \tag{6}
$$

Then apply a seven parameter Helmert transformation:

$$
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{2000.0}^{ETRF2014} = - \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} + (1-D) \begin{bmatrix} 1 & R_z & -R_y \\ -R_z & 1 & R_x \\ R_y & -R_x & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{2000.0}^{SWEREF99} \tag{7}
$$

Change the epoch to whichever the ITRF 2014 coordinates are desired in:

$$
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{t_c}^{ETRF2014} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{2000.0}^{ETRF2014} + (t_c - 2000.0) \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{NKG\_RF17vel} \tag{8}
$$

Finally, transform from ETRF 2014 to ITRF 2014:

$$
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{t_c}^{ITRF2014} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{t_c}^{ETRF2014} + (t_c - 1989) \begin{bmatrix} 0 & \dot{R}_z & -\dot{R}_y \\ -\dot{R}_z & 0 & \dot{R}_x \\ \dot{R}_y & -\dot{R}_x & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{t_c}^{ETRF2014} \tag{9}
$$

Working example

To demonstrate the transformations and help with debugging implementations, we here present an example where the coordinates of three fictive stations are transformed from ITRF 2014 to SWEREF 99. The three stations are chosen to be in the south, middle and north of Sweden, respectively. The reference epoch of their ITRF 2014 coordinates are chosen to be 2020.25. The initial ITRF 2014 coordinates are given in Table 3.

Table 3: Initial ITRF2014 coordinates (epoch 2020.25) of the example stations.

Station	X [m]	Y [m]	Z [m]
NORTH	2251700.0000	819600.0000	5891200.0000
MIDDLE	2885900.0000	827500.0000	5608600.0000
SOUTH	3468700.0000	864800.0000	5264500.0000

After the first transformation, the ETRF 2014 coordinates at epoch 2020.25 are obtained, see Table 4.

Table 4: The coordinates from table 3, transformed into ETRF 2014 (epoch 2020.25).

Station	X [m]	Y [m]	Z [m]
NORTH	2251700.5696	819599.6615	5891199.8294
MIDDLE	2885900.5477	827499.5911	5608599.7785
SOUTH	3468700.5244	864799.5276	5264499.7321

The velocities of the points, obtained from interpolating the grids, are given in Table 5.

Table 5: The velocities of the stations in the example, in the North, East, nd Up directions.

Station	V_n (mm/year)	V_e (mm/year)	V_u (mm/year)
NORTH	0.3070	-0.7819	6.3702
MIDDLE	-0.3879	-0.7701	8.8991
SOUTH	-0.5190	-0.4270	1.5240

The velocities converted to cartesian coordinates are given in Table 6.

Table 6: The velocities of the station in the example, in cartesian coordinates.

Station	V_x (mm/year)	V_y (mm/year)	V_z (mm/year)
NORTH	2.2423	-0.0159	6.0213
MIDDLE	4.5575	0.5057	7.6754
SOUTH	1.3477	-0.1041	0.9733

The resulting ETRF 2014 coordinates at 2000.0 are given in Table 7.

Table 7: The coordinates of the example stations in ETRF 2014 (epoch 2000.0).

Station	X [m]	Y [m]	Z [m]
NORTH	2251700.5241	819599.6618	5891199.7075
MIDDLE	2885900.4555	827499.5809	5608599.6231
SOUTH	3468700.4971	864799.5297	5264499.7124

After applying the seven parameter Helmert transformation, SWEREF 99 coordinates at epoch 2000.0 are obtained. These are given in Table 8.

Table 8: The coordinates of the example stations in SWEREF 99 (epoch 2000.0).

Station	X [m]	Y [m]	Z [m]
NORTH	2251700.5598	819599.6862	5891199.6497
MIDDLE	2885900.4927	827499.6118	5608599.5640
SOUTH	3468700.5356	864799.5674	5264499.6521

Finally, the coordinates in SWEREF 99 are obtained by changing the epoch to 1999.5. These are given in Table 9.

Table 9:The coordinates of the example stations in SWEREF 99 (epoch 1999.5).

Station	X [m]	Y [m]	Z [m]
NORTH	2251700.5587	819599.6862	5891199.6467
MIDDLE	2885900.4905	827499.6116	5608599.5602
SOUTH	3468700.5350	864799.5674	5264499.6517

Note, that even though the coordinates above are given with four decimals, no rounding off was done between the steps. Thus, if the coordinates from the intermediate steps are used for testing, deviations of the order of 0.1 mm may occur.

Uncertainty of the transformation

Most of the uncertainty of the transformation is in the Helmert transformation in step 3. When determining the parameters of this transformation, ITRF 2014 coordinates of the SWEREF 99 fundamental stations (epoch 2015.0) were transformed to ETRF 2014 (epoch 2000.0), and then the parameters were obtained by fitting these coordinates to their SWEREF 99 (extrapolated to epoch 2000.0) coordinates. The post-fit residuals after this fit are shown in Figure 1. An estimate of the standard uncertainty related with the Helmert transformation in step 3 can thus be obtained by looking at the RMS (Root-Mean-Square) of the residuals. These were 0.9 mm, 0.7 mm, and 2.1 mm, for the North, East, and Up directions, respectively. These values can be considered as an estimate of the uncertainty of the whole transformation when the epoch of the ITRF 2014 coordinates is 2015.0.

For epochs different from 2015.0, also the error in the NKG_RF17vel model needs to be considered. The uncertainty of the velocities from this model are about 0.1 mm/year in the horizontal directions and 0.5 mm/year in the vertical. These uncertainties need to be added quadratically to those of the

Helmert transformation to obtain a final estimate of uncertainty. For example, for the epoch 2025.0 the uncertainties would be about 1.3 mm in the horizontal directions and 5.4 mm in the vertical. In most cases, this is thus smaller than the uncertainty of the original ITRF 2014 coordinates.

Figure 1: Post-fit residuals after the least-squares fit made to obtain the Helmert transformation parameters (Table 2). The left figure shows the horizontal components and the right figure the vertical.
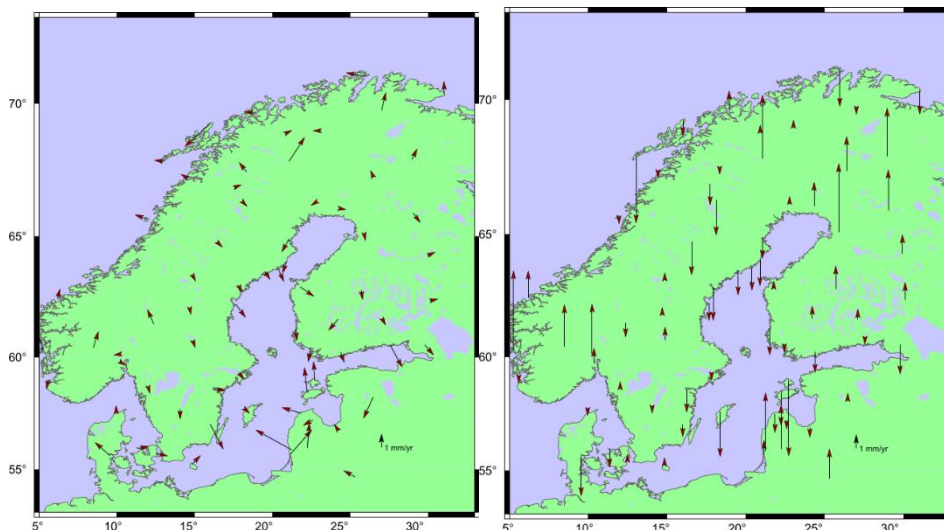
Summary and Outlook

This memo describes the transformation between ITRF 2014 and SWEREF 99. This is required, for example, for transforming the results from a global PPP service (which typically gives positions in ITRF) into SWEREF 99. The transformation described in this memo can be extended to transform coordinates in any reference frame (e.g. another ITRF) to SWEREF 99, provided that the transformation from that frame to ITRF 2014 is available. First transform the coordinates to ITRF 2014, then apply the transformation of this memo to obtain the coordinates in SWEREF 99.

Along with this transformation, also transformations between ITRF 2014 and the national reference frames of the other Nordic/Baltic countries were developed by the NKG. These transformations are similar to the one described in this memo, except that the transformation parameters of the Helmert transformation in step 3 are different and that the reference epochs differ (step 4). The transformations are implemented in the PROJ software, starting with PROJ 7.2.1.

References

Z. Altamimi, P. Rebischung, L. Métivier, X. Collilieux, ITRF2014: A new release of the International Terrestrial Reference Frame modeling nonlinear station motions, J. Geophys. Res. Solid Earth, 121(8), pp. 6109-6131, 2016, https://doi.org/10.1002/2016JB013098

Z. Altamimi, EUREF Technical Note 1: Relationship and Transformation between the International and the European Terrestrial Reference Systems, 2018, http://etrs89.ensg.ign.fr/pub/EUREF-TN-1.pdf

P. Häkli, K. Evers, L. Jivall, T. Nilsson, S. Himle, K. Kollo, I. Liepiņš, E. Paršeliūnas, O. Vestøl, M. Lidberg, NKG2020 transformation: An updated transformation between dynamic and static reference frames in the Nordic and Baltic countries, Journal of Geodetic Science, vol. 13, no. 1, 2023, pp. 20220155, https://doi.org/10.1515/jogs-2022-0155

L. Jivall, M. Lidberg, SWEREF 99 - an Updated EUREF Realisation for Sweden, Proceedings EUREF 2000 Symposium, Tromsø, 22 - 24 June 2000, http://www.euref.eu/symposia/book2000/P_167_175.pdf

ITRF Website: http://itrf.ign.fr/

EUREF Website: http://www.euref.eu/

NKG Website: https://www.nordicgeodeticcommission.com/

PROJ software: https://proj.org/